A Mathematical Modelling And 3D Simulation Of Zno Piezoelectric Base Cantilever For Pressure Sensing

Maibam Sanju Meetei, Aheibam Dinamani Singh, Swanirbhar Majumder, Ome Moyong

Abstract: The main study in this work is to derive the mathematical model of the cantilever based pressure sensor. Two steps of mathematical modelling are being applied, mechanical and electrostatic, for finding the factors affecting the output of the sensor. Two main modes of operation for piezoelectric pressure sensor are transverse and longitudinal which are described in details. For 3D structure analysis, COMSOL 5.2 multiphysics simulator is used for simulation to validate the mathematical analyses output. A comparative analysis is done for simulated output of mechanical stress and the mathematical calculated stresses of the sensor. Similarly, the simulated output voltage and mathematical calculated voltage are also analyzed. The factors affecting the sensitivity of the sensor are the length, the thickness of the mechanical structure of the sensor, thickness of the sensing material and the piezoelectric voltage coefficient (g_{31}). The output voltage of the sensor is linearly vary with the input pressure with negative slope. The sensitivity of the sensor for simulated is -0.012 mV/Pa and mathematical is -0.017 mV/Pa.

Index Terms: moment of inertia, operation mode, piezoelectric voltage-coefficient, stress.

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1. INTRODUCTION

There are different types of pressure sensors viz. capacitive, inductive, piezo-resistive, piezo-electric, electromagnetic, optical, resonance etc. Among these sensors, piezoelectric is a passive sensor which doesn't require any external excitation. Piezoelectric based pressure sensor can be implemented to MEMS sensor. Various applications of MEMS pressure sensors are Automotive application: Tire Pressure Monitoring Systems (TPMS), Gasoline Particulate Filter (GPF), Exhaust Gas Recirculation (EGR), Fuel/Oil Pressure, Air Brakes, Medical Applications: Blood Pressure Monitoring, Continuous Positive Airway Pressure (CPAP), Dialysis Machines, Negative Pressure Wound Therapy, Industrial Applications: Safety Cabinets/Ventilation Hoods, Gas Flow Instrumentation, Liquid Level Measurement, Hydraulic & Pneumatic Pressure, Oil Filled Media Isolated Sensors, Heating Ventilation Air Conditioning (HVAC) Applications: Variable Air Volume (VAV), Fire Pressurization and Smoke Management, Filter Monitoring, Compressors, Consumer Applications: Washing Machines, Vacuum Cleaner, Coffee & Espresso Machines, Water Purifiers, Invasive Medical Applications: Therapeutic Catheter, Endoscope, Robots Application: Robotics Hands, Robotics Legs[1][2][3][4].

In beam structure there are two common structures used in sensor design. They are cantilever and bridge structures. Cantilever structures are clamped at one terminal whereas bridge structures are clamped at two terminals. For a cantilever beam, the maximum deflection is at the free terminal but the maximum stress is at the edges of the fixed clamped side [5]. For the bridge structure, the maximum deflection is at the center of the bridge but the stress is occurring at both the edges of the fixed terminals. In cantilever, the two most common way of loading of force are the uniform force or pressure which is load through the surface and the fixed point which is loaded at the free end of the cantilever. In mono-morph cantilever, the placement of the sensing layer is done near the fixed edge of the top surface and in bi-morph cantilever the placement of the sensing layer are done near the fixed edge of top and bottom surface of the cantilever. These placements of the sensing layers are done near the fixed edge because the maximum stress occurs at that place. The output of the piezoelectric is directly proportional to the stress. Recently, many researchers developed Zincoxide (ZnO) thin film deposition for various applications. ZnO has a wide application in the filled of semiconductor, photoelectric, photoconductivity [6], energy harvesting [7] and pressure sensor [8]. This study emphasizes on the mathematical modelling and simulation of a cantilever piezoelectric pressure sensor for low pressure. The design model of a cantilever beam structure with ZnO as sensing layer was simulated for applied pressure ranging from 0 to 20 Pa with 2 Pa step size. For improving the output, the thickness of the sensing material may be increased and for the sensor structure, the length and the height should be consider according to the mathematical derived equation. This paper has three main parts. First, explains the mathematical modelling of the sensor which the mechanical modelling and electrostatic modelling are described. Second, explains the design structure and 3D analysis of the designed sensor structure with COMSOL multiphysics simulator. Third, after various comparative analysis of the mathematical analyses outputs and 3D analyses outputs conclude the various factor affecting the sensitivity and the behavior of the designed sensor.

2. SENSOR MODELLING

For a cantilever beam structure pressure sensor based on piezoelectric can be model under two stages.

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2.1 Mechanical modelling

In this modelling, consider a uniform load or pressure is applied downward on the top of the cantilever surface as shown in figure 1.

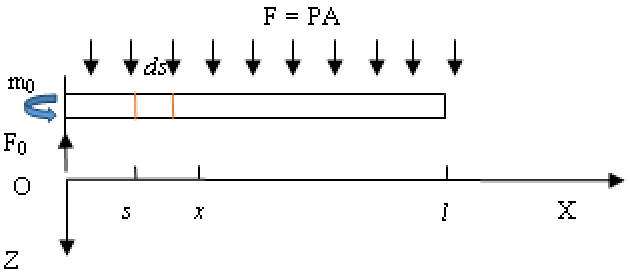

***Fig. 1**: Structure of cantilever load with uniform force.*

In [5], the generalized governing equation for the cantilever structure is given by equation no. 1.

$$-EI\frac{\partial^2 w}{\partial x^2} = Fx - m_0 - \int_0^x \frac{F}{l}(x-s)\partial s\,. \tag{1}$$

Where, E is the Young's Modulus, I is moment of inertia, $\frac{\partial^2 w}{\partial x^2}$ is the double derivative of deflection, F is the uniform force applied, x is the distance from the origin, m_0 is the moment at the hinges, l is the length of the beam, s is the position on a beam, b is the width of the beam, h is the thickness of the beam.

Integrating equation 1 with respect to s we get,

$$-EI\frac{\partial^2 w}{\partial x^2} = Fx - m_0 - \frac{F}{l}(sx - \frac{s^2}{2})_0^x\,.$$

(2.1)

$$-EI\frac{\partial^2 w}{\partial x^2} = Fx - m_0 - \frac{F}{l}(x^2 - \frac{x^2}{2})\,.$$

(2.2)

$$-EI\frac{\partial^2 w}{\partial x^2} = Fx - m_0 - \frac{Fx^2}{2l}\,.$$

(2.3)

Now,

$$\frac{\partial^2 w}{\partial x^2} = \left[\frac{Fx^2}{2l} + m_0 - Fx\right]\frac{1}{EI}\,. \tag{3}$$

The equation of the stress T(x) is given by equation 4.

$$T(x) = -E(-\frac{h}{2})\frac{\partial^2 w}{\partial x^2}\,. \tag{4}$$

Putting the value of $\frac{\partial^2 w}{\partial x^2}$ from equation 3 in the equation (4) of stress we get,

$$T(x) = -E(-\frac{h}{2})\left[\frac{Fx^2}{2l} + m_0 - Fx\right]\frac{1}{EI}\,. \tag{5}$$

Putting x = l and the boundary condition $\left.\frac{\partial^2 w}{\partial x^2}\right|_{l=0} = 0$ in equation 2.3 we get,

$$0 = Fl - m_0 - \frac{Fl^2}{2l}\,. \tag{6}$$

Therefore, the value of m_0 is given by the equation 7.

$$m_0 = \frac{Fl}{2}\,. \tag{7}$$

Putting the value of m_0 in the equation 5, the value of stress is given by

$$T(x) = -E(-\frac{h}{2})\left[\frac{Fx^2}{2l} + \frac{Fl}{2} - Fx\right]\frac{1}{EI}\,.$$

(8.1)

$$T(x) = \frac{h}{2I}\left[\frac{Fx^2}{2l} + \frac{Fl}{2} - Fx\right].$$

(8.2)

$$T(x) = \frac{h}{4Il}\left(Fl^2 + Fx^2 - 2Fxl\right).$$

(8.3)

The maximum stress is at x = 0, the equation 8.3 will be

$$T_{\max} = \frac{h}{4Il}Fl^2\,. \tag{9}$$

Putting the value of moment of inertia, $I = \frac{bh^3}{12}$ in equation 9, the expression of maximum stress is given by equation

$$T_{\max} = \frac{h}{4l\frac{bh^3}{12}}Fl^2\,. \tag{10.1}$$

$$T_{\max} = \frac{3}{bh^2}Fl$$

(10.2)

Putting the value of force, F = PA in equation 10.2 the maximum stress is given by the following equation

$$T_{\max} = \frac{3Pl^2}{h^2} \tag{11}$$

Where, F is the force applied, P is the pressure and A is the area. Area (A) = Length (l) x Breadth (b).

2.2 Electrostatic Modelling

In this modelling, let us consider the two most common mode of operation [1]. First, the transverse mode of operation which the direction of the induced voltage is along the direction of the applied force but perpendicular to the induced strain. Second, longitudinal mode of operation which the direction of the induced voltage is along the direction of the strain. Now, in case of cantilever beam structure, the operational mode of the sensor comes under the transverse mode (d_{31}). The output voltage on the surface plates of the sensor is given by the following equation 12 [9].

$$V(x) = g_{31}T(x)t. \tag{12}$$

Where, g_{31} is the voltage coefficient of the piezoelectric material, T(x) is the stress on the surface, t is the thickness of the piezoelectric and V(x) is the induced voltage.

Sensitivity: As the definition of the sensitivity is the ratio of the change in the output to the change in the input. Thus,

$$Sensitivity(S) = \frac{\Delta output}{\Delta input} = \frac{\delta V}{\delta P} \text{ V/Pa} \tag{13.1}$$

$$Sensitivity(S) = \frac{g_{31}t(T(x)_2 - T(x)_1)}{P_2 - P_1}. \tag{13.2}$$

Where, $T(x)_2$ is the stress on the surface when applying pressure P_2 and $T(x)_1$ is the stress on the surface when applying pressure P_1. The stress T(x) of the cantilever is directly proportional to the length and applied pressure is inversely proportional to the thickness of the cantileverThus, the sensitivity is depended upon the length (l) of the cantilever, voltage coefficient (g_{31}) of the piezoelectric material, thickness (t) of the piezoelectric material and inversely proportional to the thickness (h) of the sensor.

3 DESIGN AND SIMULATION

A sensor model has been design in the Comsol Multiphysics simulator for validating the mathematical model of the sensor. A cantilever beam structure consisting of silicon (Si) layer as main mechanical structure. Silicon dioxide (Sio_2) is used as the insulating layer between semi-conductor (Si) and metal electrode Gold (A_u). A piezoelectric layer of ZnO is taken in between the two electrodes which will act as a sensing material. A block diagram of the sensor has been shown in the fig. 2. The physical dimension of the sensor is tabulated at the table 1. The necessary properties of the sensor materials are tabulated in table 2.

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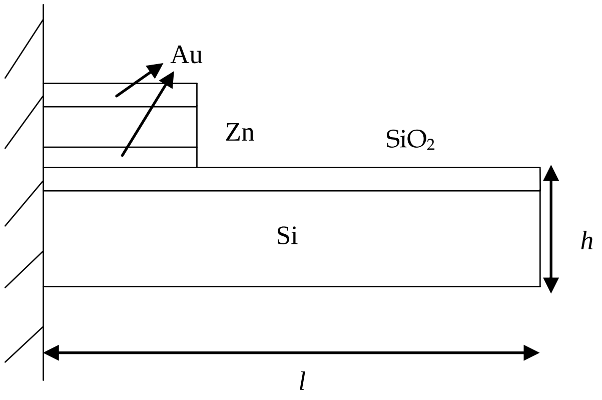


***Fig. 2**: Block diagram of sensor.*

TABLE 1
THE PHYSICAL DIMENSION OF THE SENSOR

Material	Length (l) µm	Breadth (b) µm	Height (h) µm
Si	l_{Si} 300	40	50
SiO_2	l_{SiO2} 300	40	3
Au	l_{Au} 40	40	2
ZnO	l_{ZnO} 40	40	5

TABLE 2
MATERIAL PROPERTIES OF THE MATERIAL USED IN THE SENSOR

Material	Young's Modulus (E)	Poission's ratio (v)	Voltage coefficient (g_{31})
Si	160×10^9 Pa	0.22	--
SiO_2	70×10^9 Pa	0.17	--
Au	70×10^9 Pa	0.44	--
ZnO	120 Pa	0.46653	-4.85×10^{-2} Vm/N

Various Comparative analysis of 3D simulated output values and mathematical derive equation values are carried out for verification of the mathematical modelled equations. As in the design there are two parts, first, the mechanical and the electrostatic. Thus, a comparative study for mechanical and electrostatic are analyzed separately.

3.1 Mechanical analysis

In this, the maximum stress of the cantilever for a simulated and mathematical derived values are tabulated as in table 3 and the graphical representation is fig. 4 for different values of applied pressure. The output of the simulated is shown in fig 3 which is showing stress distribution of the cantilever for an applied pressure of 20 Pa. This figure also shows the maximum stress is occurred at the fixed end side of the cantilever.

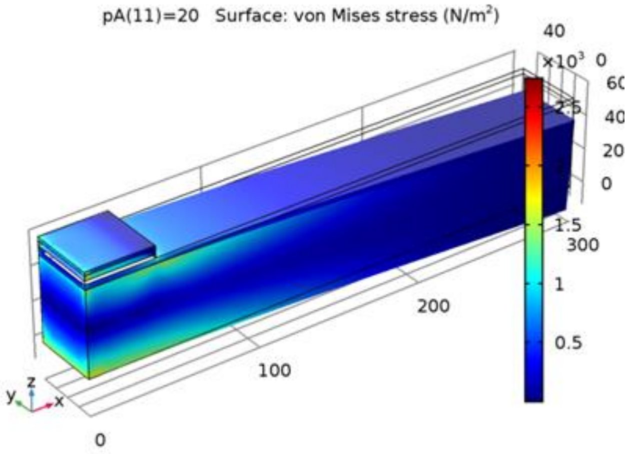


***Fig. 3**: Simulated output stress.*

In fig. 4, the maximum output stresses are varied linearly to the applied pressures with positive slope. The simulated outputs values and the mathematical mechanical modelling values are closed to each other.

TABLE 3
COMPARATIVE STUDY OF MAXIMUM STRESS FOR METHEMATICAL AND SIMULATED VALUE.

Applied Pressure (P_a)	Mathematical Maximum Stress (Pa or N/m^2)	Simulated Maximum Stress (Pa or N/m^2)
2	140.00	115.51
4	280.95	226.05
6	421.43	355.04
8	561.91	468.79
10	702.39	585.64
12	842.87	701.13
14	983.35	824.45
16	1123.80	928.03
18	1264.30	1030.01
20	1404.00	1160.21

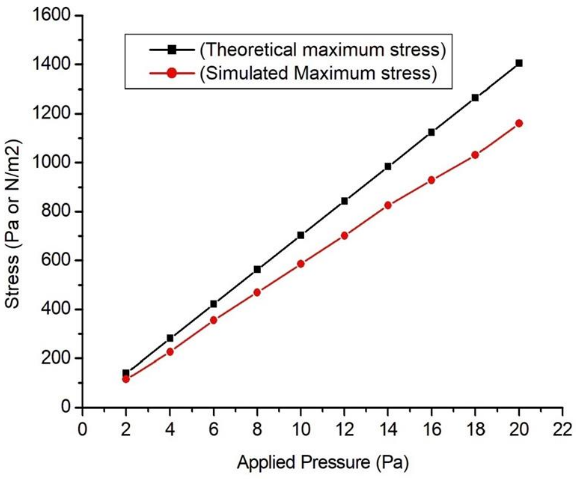


***Fig.** 4: Pressure vs Maximum stress output.*

3.2 Electro-statically analysis

In this, the voltage of the cantilever for a simulated and mathematical derived are tabulated as in table no. 4 and the graphical representation of the table in fig. 6. The output of the simulated is shown in fig. 5. This figure shows the voltage distribution on the surface of the piezoelectric for applied pressure at 20 Pa. the output voltage shows higher magnitude where the stress is higher but negative values.

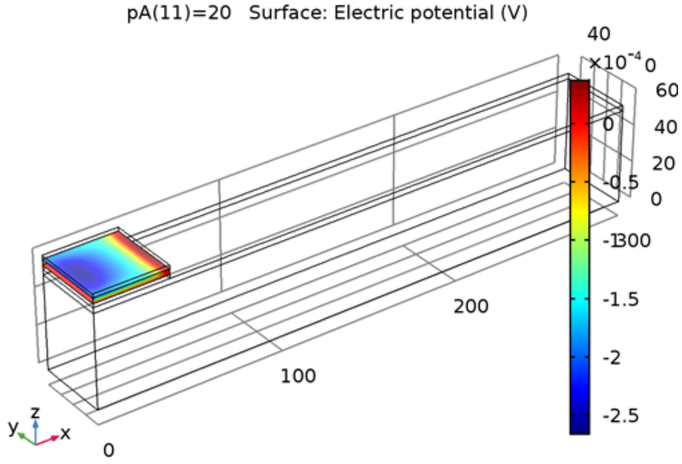
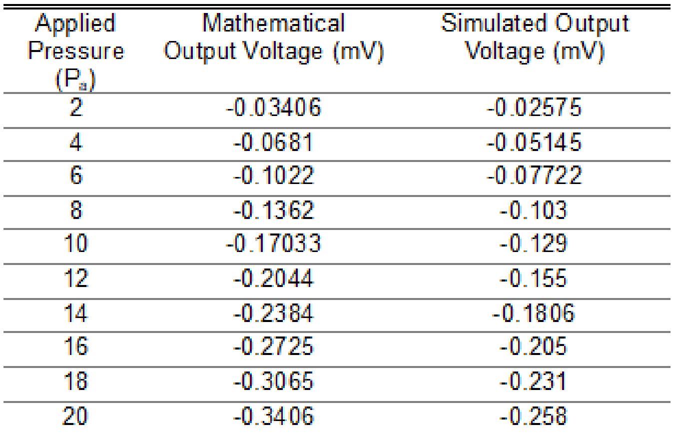


***Fig. 5**: Simulated output voltage.*

From the table 4 and fig. 6, the output voltages are negative value but increase magnitude with increase in pressure. The output values are linearly varying with increase in magnitude but with negative slope. The 3D simulated value and the mathematical derived values are very close to each other and showing close relation of the outputs. The sensitivity of the sensor is 0.017 mV/Pa for mathematical analysis and 0.012 mV/Pa for the simulated analysis.

TABLE 4
COMPARATIVE STUDY OF OUTPUT VOLTAGE FOR METHEMATICAL AND SIMULATED VALUE



Applied Pressure (P_a)	Mathematical Output Voltage (mV)	Simulated Output Voltage (mV)
2	-0.03406	-0.02575
4	-0.0681	-0.05145
6	-0.1022	-0.07722
8	-0.1362	-0.103
10	-0.17033	-0.129
12	-0.2044	-0.155
14	-0.2384	-0.1806
16	-0.2725	-0.205
18	-0.3065	-0.231
20	-0.3406	-0.258

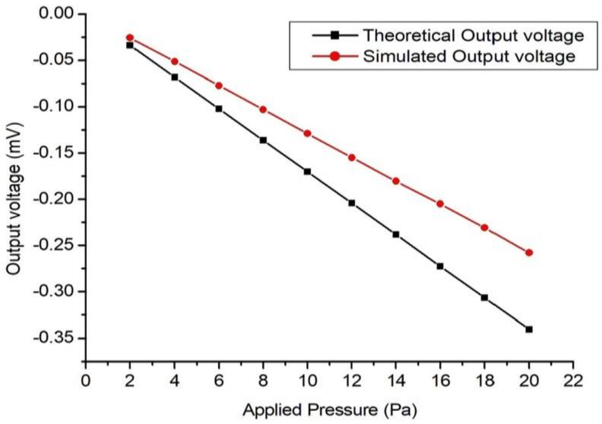


Fig 6: *Pressure (Pa) vs. Output Voltage.*

4. CONCLUSION

We have discussed, the mathematical modelling and the 3D structure analysis, designed approach for piezoelectric based cantilever pressure sensor. The simulation is done in COMSOL multiphysics simulator for ZnO base cantilever pressure sensor. The mathematical modelling of the stress and the output voltage of the cantilever for an applied pressure is developed. For Validation of mathematical modelling, various comparative studies are carried out between the mathematical analyses values and the simulated values for different applied pressures. The various factor affecting the sensitivity are the physical dimension of the sensor, the voltage coefficient of the piezoelectric material and the operational mode of the sensor. The negative voltage is observed where the tensile stress is occurred and positive voltage is observed at where the compressive stress is occurred. The input pressure is directly proportional to the mechanical stress with positive slope and the output voltage with negative slope. The sensitivity of the sensor is 0.017 mV/Pa for mathematical analysis and 0.012 mV/Pa for the simulated analysis.

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