On Completely And Semi Completely Prime Ideal With Respect To An Element Of A Boolean α_1,α_2 Near-Ring

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Abstract: In this paper ,I generalize concepts of α_1,α_2 near-ring as well as of completely semi prime ideal with respect to an element x of a α_1,α_2 near-ring and the completely prime ideal of a α_1,α_2 near-ring with respect to an element x and the relationships between the completely prime ideal with respect to an element x of a α_1,α_2 near-ring and some other types of ideals,as well as I will valid if I is c.p.I of α_1,α_2 near-ring iff is a x-c.s.p.I of a α_1,α_2 near-ring as well as valid if I is c.p.I of α_1,α_2 near-ring iff is a x-c.s.p.I of α_1,α_2 near-ring as well as I is e'-c.p.I of α_1,α_2 near-ring iff it is a e'-c.s.p.I of α_1,α_2 near-ring and all its valid after put condition of Boolean a α_1,α_2 near-ring

Keywords :α_1,α_2 near-ring,c.s.p.I,c.p.I,x-c.s.p.I,x-c.p.I

1 INTRODUCTION

In 1905 L.E Dickson began the study of near- ring and later 1930 Wieland has investigated it .Further, material about a near ring can be found (cf.[2,5,9]).In 1977 G. Pilz, was introducing the notion of prime ideals of a near-ring (cf.[1,2,7]).. In 1988 N.J.Groenewald was introducing the notions of completely (semi) prime ideals of a near-ring (cf.[6,10]). In 2011 H.H.abbass, S.M.Ibrahem introduced the concepts of a completely semi prime ideal with respect to an element of a near-ring[4] .In 2012 H,H.abbass,Mohanad Ali Mohammed introduce the concept of a completely semi prime ideals near-ring with respect to an element of a near-ring [3]. In2010,S.Uma,R.Balakrishnan,T.Tamizh Chelvam introduce the concept of a α_1,α_2 near-ring near- ring[8].

1.1 definition

A right near-ring is a set N together with two binary operations + and . such that

1. (N,+) is a group (not necessarily abelian)
2. (N, .) is a semigroup
3. $n_1.(n_2+n_3) = n_1 . n_2 + n_1 . n_3$,$\forall n_1,n_2,n_3 \in N$

1.2 definition:

Let N be a right near-ring , if

1-for every a in N there exists x in N such that a=xax then we say N is an α_1 near-ring.

2- for every a in N-{0} there exists x in N-{0} such that x=xax then we say N is an α_2 near-ring.

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1.3 example:

1-the near-ring (N,+,.) define on the klein's four group N={0,a,b,c} where addition and multiplication is defined as:

+	0	a	b	c		.	0	a	b	c
0	0	a	b	c		0	0	0	0	0
a	a	0	c	b		a	0	0	a	a
b	b	c	0	a		b	0	a	b	b
c	c	b	a	0		c	0	a	c	c

Is an α_1 near-ring (since bab=ab=a ,bbb=b ,ccc=c ,a0a=0) its neither α_2 near-ring (since there is no x in N-{0} such that xax=x) 2-the near-ring (N,+,.) define on the klein's four group N={0,a,b,c} ,where addition and multiplication is defined as:

+	0	a	b	c		.	0	a	b	c
0	0	a	b	c		0	0	0	0	0
a	a	0	c	b		a	a	a	a	a
b	b	c	0	a		b	0	0	b	0
c	c	b	a	0		c	a	a	c	a

Here (N,+,.) is an α_2 near-ring it is neither α_1 near-ring (since xcx≠c for any x in N).

1.4 definition:

Let N be a α_1,α_2 near-ring , normal subgroup I of (N,+) is called an right ideal of N if :

1-$NI \subseteq I$,Where $NI=\{n.i : n \in N , i \in I \}$

2- $\forall n,n_1 \in N$ and for all $i \in I$,$(n_1+i)n-n_1n \in I$

1.5 example:

Consider the a N be a α_1,α_2 near-ring example(1.3) the normal subgroup I={0,a} is ideal of the near-ring N .

1.6 Remark

We will refer that all ideals in this paper are right ideals.

1.7 definition:
Let N be a α_1,α_2 near-ring ,an ideal I of N is called a completely semi prime ideal (c.s.p.I) of N if $x^2\in I$ implies $x\in I$ for all $x\in N$.

1.8 example:
Let N be a α_1,α_2 near-ring ,the ideal I={0,a} of N in example(1.3) is a completely semi prime ideal of N, since $\forall y\in N$ implies $y\in I$.

1.9 definition:
Let N be a α_1,α_2 near-ring, an ideal of N is called a prime ideal if every ideal I_1,I_2 of N such that $I_1.I_2\subseteq I$ implies $I_1\subseteq I$ or $I_2\subseteq I$.

1.10 example:
Let N be a α_1,α_2 near-ring ,in example(1.3) the ideal I={0,a} of the N is a prime ideal .

1.11 definition:
Let N be a α_1,α_2 near-ring , s be an ideal in N so s is semi prime if and only if for all ideals I of N , $I^2\subseteq s$ implies $I\subset s$.

1.12 definition:
Let N be a α_1,α_2 near-ring,I be an ideal of N ,then I is called a completely prime ideal of N if for all $x,y\in N$,$x.y\in I$ implies $x\in I$ or $y\in I$ denoted by (c.p.I) of N .

1.13 example
Let N be a α_1,α_2 near-ring,in example(1.3) the ideal I={0,a} is complete prime ideal of the N.

1.14 proposition:
Let N be a α_1,α_2 near-ring ,every c.p.I of N is a c.s.p.I of N.

Proof:
Let for all $y\in N$, such that

$y^2\in I$ as I is c.p.I of N

So,$y^2=y.y\in I$

then,$y\in I$

so,I is a c.s.p.I of N.

1.15 definition:
Let N be a α_1,α_2 near-ring ,is called Boolean if for all $x\in N,x^2=x$.

1.16 example:
the near ring (N,+,.) define on the klein's four group N={0,a,b,c}, where addition and multiplication is defined as :

+	0	a	b	c		.	0	a	b	C
0	0	a	b	c		0	0	0	0	0
a	a	0	c	b		a	a	a	a	a
b	b	c	0	a		b	0	0	b	b
c	c	b	a	0		c	a	a	c	c

Is a α_1,α_2 near-ring,as every Boolean near ring is α_1 near-ring as well as α_2 near-ring since(000=00=0,aaa=aa=a,bbb=bb=b,ccc=cc=c).

2 Completely and semi completely prime ideal with respect to an element of a α_1,α_2 near-ring

2.1 definition:
Let N be a α_1,α_2 near-ring and $x\in N$,I is called completely semi prime ideal with respect to an element x denoted by (x-c.s.p.I) of N if for all $y\in N$, $x.y^2\in I$ implies $y\in I$.

2.2 example:
Consider the near ring N={0,a,b,c},where addition and multiplication is defined as :

+	0	a	b	c		.	0	a	b	c
0	0	a	b	c		0	0	0	0	0
a	a	0	c	b		a	0	a	0	a
b	b	c	0	a		b	0	0	b	b
c	c	b	a	0		c	0	a	b	c

is a α_1,α_2 near-ring,the ideal I={0,b} is a-c.s.p.I of N .

2.3 definition:
Let N be a α_1,α_2 near-ring, I be an ideal of N and $x\in N$, then is called a completely prime ideal with respect to an element x denoted by (x-c.p.I) of N if for all $y,z\in N$, $x.y.z\in I$ implies $y\in I$ or $z\in I$.

2.4 example:
Let N be a α_1,α_2 near-ring,in example(2.2) the ideal I={0,b} is a-c.p.I of N.

2.5 proposition:
Let N be a α_1,α_2 near-ring and $x\in N$,then every completely prime ideal with respect to an element x of N is a completely semi prime ideal with respect to an element x of N.

Proof:

let I be a x-c.p.I and if for all $y\in N$ such that $x.y^2\in I$,

$x.y^2=x.y.y\in I$

implies $y\in I$

then,I is x-c.s.p.I of N.

2.6 Remark:
The converse of the proposition (2.5) may be not true , just where put the condition of Boolean of N its valid (proposition(2.11)) .

2.7 proposition:
Let N be a Boolean α_1,α_2 near-ring then I be c.p.I of N iff I is an x-c.s.p.I for all $x\in N-I$.

Proof:
($\Rightarrow$) Let $x,y \in N$ such that $x.y^2 \in I$
as N is Boolean, then
$x.y^2 \in I \rightarrow x.y \in I$
so $y \in I$ as I is c.p.I of N
then ,I is an x-c.s.p.I for all $x \in N$-I
($\Leftarrow$) let $\forall y \in N$, $x.y \in I$
then , $x.y^2 \in I$ so that $y \in I$
then I is c.p.I of N .

2.8 proposition:
Let N be a Boolean α_1,α_2 near-ring ,then I be c.p.I of N iff I is an x-c.p.I for all $x \in N$.

Proof:
($\Rightarrow$) as I is c.p.I of α_1,α_2 near-ring
Let ,$\forall y \in N$,$x.y.y \in I$
then,$\forall y \in N$,$x.y^2 \in I$
as N is Boolean, then
$\forall y \in N, x.y \in I$ and so $y \in I$ as I is c.p.I of N
then,I is x-c.p.I of N .
($\Leftarrow$) let $\forall y \in N$, $x.y \in I$
as N is Boolean, then
$x.y^2 \in I \rightarrow x.y.y \in I$
so $y \in I$,as I is an x-c.p.I of N
then, I is c.p.I of N .

2.9 Proposition:
Let N be a Boolean α_1,α_2 near-ring ,then every x-c.p.I of N is c.s.p.I of N.

Proof:
As every x-c.p.I of N is c.p.I of N (by proposition (2.8)) and every c.p.I of N is c.s.p.I of N (by proposition (1.14)) then every x-c.p.I of N is c.s.p.I of N.

2.10 Proposition:
Let N be a Boolean α_1,α_2 near-ring , then every x-c.s.p.I of N is c.s.p.I of N.

Proof:
As every x-c.s.p.I of N is c.p.I of N (by proposition (2.7)) and every c.p.I of N is c.s.p.I of N (by proposition (1.14)) then every x-c.s.p.I of N is c.s.p.I of N.

2.11 Proposition:
Let N be a Boolean α_1,α_2 near-ring , then every x-c.s.p.I of N is x-c.p.I of N.

Proof:
As every x-c.s.p.I of N is c.p.I of N (by proposition (2.7)) and every c.p.I of N is x-c.p.I of N (by proposition (2.8)) then every x-c.s.p.I of N is x-c.p.I of N.

2.12 Remark:
The following diagram show us the relationships between these ideals

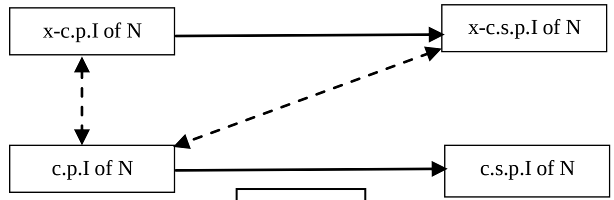

Figure (1)

3 Completely and semi completely prime ideal α_1,α_2 near-ring with respect to an element

3.1 definition:
Let N be a α_1,α_2 near-ring , is called a completely semi prime ideal near ring, denoted by (c.s.p.I α_1,α_2 near-ring) if every ideal of N are c.s.p.I of N .

3.2 example:
Let N be a α_1,α_2 near-ring , in example(2.2) is a c.s.p.I of N since all its ideals $I_1=\{0,a\}$, $I_2=\{0,b\}$, $I_3=\{0,a,b\}$, $I_4=\{0\}$,$I_5=N$ are c.s.p.I of N.

3.3 definition:
Let N be a α_1,α_2 near-ring , is called a completely prime ideal near ring, denoted by (c.p.I α_1,α_2 near-ring) if every ideal of N are c.p.I of N.

3.4 example:
Let N be a α_1,α_2 near-ring , in example(2.2) is a c.p.I of N since all its ideals $I_1=\{0,a\}$, $I_2=\{0,b\}$, $I_3=\{0,a,b\}$, $I_4=\{0\}$,$I_5=N$ are c.p.I of N.

3.5 proposition:
Let N be a α_1,α_2 near-ring ,if N is a c.p.I α_1,α_2 near-ring ,then N is a c.s.p.I α_1,α_2 near-ring .

Proof:
let N is a c.p.I α_1,α_2 near-ring so, every ideal of N is a c.p.I of N then ,every ideal of N is a c.s.p.I of N (by proposition(1.14)) then ,N is a c.s.p.I α_1,α_2 near-ring .

3.6 definition:
Let N be a α_1,α_2 near-ring , is called a x- completely semi prime ideal α_1,α_2 near-ring ,denoted by (x-c.s.p.I α_1,α_2 near-ring) if every ideal of N are x-c.s.p.I of N, where $x \in N$.

3.7 example:
Let N be a α_1,α_2 near-ring, in example(2.2) is a (c-c.s.p.I of N) since all its ideals $I_1=\{0,a\}$, $I_2=\{0,b\}$, $I_3=\{0,a,b\}$, $I_4=\{0\}$,$I_5=N$ are c-c.s.p.I of N.

3.8 definition:
Let N be a α_1,α_2 near-ring is called a x- completely prime ideal α_1,α_2 near-ring ,denoted by (x-c.p.I α_1,α_2 near-ring) if every ideal of N are x-c.p.I of N, where $x \in N$.

3.9 example:
Let N be a α_1,α_2 near-ring in example(2.2) is a (c-c.p.I of N) since all its ideals $I_1=\{0,a\}$, $I_2=\{0,b\}$, $I_3=\{0,a,b\}$, $I_4=\{0\}$,$I_5=N$ are c-c.p.I of N.

3.10 proposition:
If N is a x-c.p.I α_1,α_2 near-ring ,then N is a x-c.s.p.I α_1,α_2 near-ring,where $x\in N$.

Proof:
Let N is a x-c.p.I α_1,α_2 near-ring so, every ideal of N is a x-c.p.I of α_1,α_2 near-ring then, every ideal of N is a x-c.s.p.I of α_1,α_2 near-ring (by proposition(2.5)) then, N is a x-c.s.p.I α_1,α_2 near-ring .

3.11 proposition:
Let N be a Boolean α_1,α_2 near-ring then N be c.p.I iff N is an x-c.s.p.I for all $x\in N$-I .

Proof:
$\Rightarrow$) let N is c.p.I of α_1,α_2 near-ring (so, every ideal of N is a c.p.I of N then, every ideal of N is a x-c.s.p.I of α_1,α_2 near-ring(by proposition(2.7)) then, N is an x-c.s.p.I for all $x\in N$-I($\Leftarrow$) let N is an x-c.s.p.I for all $x\in N$-I so, every ideal of N is a x- c.s.p.I of N then, every ideal of N is a c.p.I of N (by proposition(2.7)) then N be c.p.I

3.12 proposition:
Let N be a Boolean α_1,α_2 near-ring then N be c.p.I of N iff N is an x-c.p.I for all $x\in N$.

proof:
Obvious by definitions above and proposition (2.8)

3.13 Proposition:
Let N be a Boolean α_1,α_2 near-ring ,if N is x-c.p.I then N is c.s.p.I

Proof:
Obvious by definitions above and proposition (2.9)

3.14 Proposition:
Let N be a Boolean α_1,α_2 near-ring ,if N is x-c.s.p.I then N is c.s.p.I

Proof:
Obvious by definitions above and proposition (2.10)

3.15 Proposition:
Let N be a Boolean α_1,α_2 near-ring ,if N is x-c.s.p.I then N is x-c.p.I .

Proof:
Obvious by definitions above and proposition (2.11)

3.16 Remark:
The following diagram show us the relationships between these a α_1,α_2 near-ring

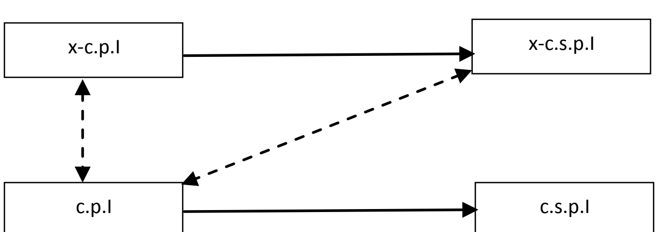

Figure (2)

4 Completely and semi completely prime ideal with respect to an multiplicative identity of a α_1,α_2 near-ring

4.1 proposition:
Let N be a Boolean α_1,α_2 near-ring with multiplicative identity e' then I is e'-c.s.p.I of N iff it is a c.s.p.I of N .

4.2 proposition:
Let N be a Boolean α_1,α_2 near-ring with multiplicative identity e' then I is e'-c.p.I of N iff it is a c.p.I of N .

4.3 proposition:
Let N be a Boolean α_1,α_2 near-ring with multiplicative identity e' then I is e'-c.p.I of N iff it is a e'- c.s.p.I of N .

Proof:
($\Rightarrow$) let $\forall y\in N$,$x.y^2\in I$
as I is e'-c.p.I of N
let $x.y.y\in I$,then
$e'.x.y.y\in I$,then
$e'.x.y.y=e'.x.y^2=x.y^2\in I$
then ,$y\in I$ as I is e'-c.p.I of N
so , I is a e'- c.s.p.I of N
($\Leftarrow$) let $\forall y\in N$,$x.y.y\in I$
as ,I is a e'- c.s.p.I of N
Let $x.y^2\in I$,then
$e'.x.y^2\in I$,then
$e'.x.y^2=e'.x.y.y=x.y.y\in I$
then, $y\in I$ as I is a e'- c.s.p.I of N
so,I is e'-c.p.I of N .

4.4 example:
Let N={0,a,b,c} be a α_1,α_2 near-ring ,with addition and multiplication defined as:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	a	0
b	0	a	b	c
c	0	0	c	c

the ideal I={0,a} is b-c.p.I of N iff it is a b- c.s.p.I of N,but its same example which clear that not all c.p.I of N are x-c.p.I for all $x\in N$ since I={0,a} is c.p.I of N but it is not a-c.p.I of N ,since $a.(b.c)=0\in I$ but $b\notin I$ and $c\notin I$ and not all c.s.p.I of N are x-c.s.p.I of N since I={0,a} is c.s.p.I of N but it is not a-c.s.p.I of N ,since $a.b^2\in I$ but $b\notin I$.

4.5 proposition:
Let N be a Boolean α_1,α_2 near-ring with multiplicative identity e' then I is c.p.I of N iff it is a e'- c.s.p.I of N .

4.6 Remark:
The following diagram show us the relationships between these ideals

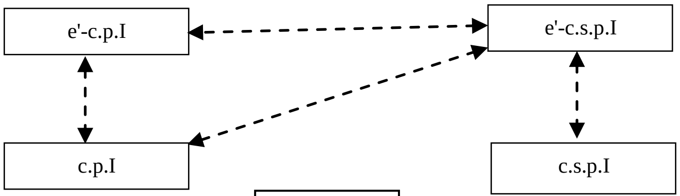

Figure (3)

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