An Optimum Stratification For Stratified Cluster Sampling Design When Clusters Are of Varying Sizes

Shikha Mehta, V. L. Mandowara

Abstract—The paper considers the problem of determining optimum strata boundaries for cluster sampling design considering unequal sizes of clusters. The minimal equations giving optimum strata boundaries by minimising the variance of the estimator of the population mean. sampling in each stratum being carried out independently by simple random sampling without replacement (SRSWOR). These minimal equations are difficult to solve exactly. Thus, the approximate solutions to these minimal equations have been obtained for three allocation methods namely proportional, equal and neyman allocation. The paper concludes with Numerical illustrations.

Index Terms – Approximate Solutions, Minimal Equations, Optimum Strata Boundaries, SRSWOR, Proportional, Equal and Neyman allocation.

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1 INTRODUCTION

The problem of optimum stratification for single stage design was first considered by Dalenius (1950) later on many contributions by Dalenius and Gurney (1951), Dalenius and Hodges (1957), Ghosh (1963), Taga (1967), Serfling (1968), Singh and Sukhatme (1969), Gupta and Seth (1979). Mandowara and Gupta (1994, 1999) were first to extended these results for two stage and multi stage setup with equal sizes of p.s.u.(primary stage units) and sub- sequent units. Rajyaguru and Gupta (1995) have suggested an alternative aspect of optimum stratification. Vinita and Gupta (2005) have taken-up the problem of optimum stratification of various estimators for mean and pooled variance in cluster design. The present paper concerns with optimum points of stratification for cluster sampling design using different allocation methods when clusters are of varying sizes.

2 SAMPLING DESIGN

Let there be a finite population π of size N clusters which is divided into L non overlapping strata with h^{th}stratum consisting of N_h clusters $(h = 1,2,\ldots\ldots L)$ and i^{th}cluster consisting of M_{hi} elements where $(j = 1,2,\ldots\ldots M_{hi}, i = 1,2,\ldots\ldots N_h, h = 1,2,\ldots\ldots L)$. Let Y_{hij} be the value of character Y on the j^{th}element in the i^{th}cluster of h^{th}stratum. The population mean is given by

$$\overline{Y} = \sum_{h=1}^{L} \lambda_h \overline{Y}_{h..} \qquad (2.1)$$

Where

$\sum_{h=1}^{L} N_h = N$ the total number of clusters in the population

$\lambda_h = \frac{M_{h0}}{M_0}$ proportion of elements falling in the h^{th}stratum

$M_{h_0} = \sum_{i=1}^{N_h} M_{hi}$ be the total no. of elements in the h^{th}stratum

$M_0 = \sum_{h=1}^{L} M_{h_0}$ be the total no. of elements in the population

$\overline{Y}_{h..} = \frac{1}{M_{h0}} \sum_{i=1}^{N_h} \sum_{j=1}^{M_{hi}} Y_{hij}$ be the population mean of h^{th} stratum

To estimate the population mean, we select independent samples of $n_h$clusters from h^{th}stratum. Sampling in each stratum being carried out independently by simple random sampling without replacement (SRSWOR). An unbiased estimate of population mean (2.1) is given by

$$\bar{y}_w = \sum_{h=1}^{L} \lambda_h \bar{y}_{h..} \qquad (2.2)$$

Where

$\sum_{h=1}^{L} n_h = n$ the total number of clusters in the sample.

$\bar{y}_{h..} = \frac{1}{n_h} \sum_{i=1}^{n_h} \frac{M_{hi}}{\overline{M}_h} \bar{y}_{hi}$ is the sample mean for h^{th} stratum.

$\overline{M}_h = \frac{M_{h0}}{N_h}$ is the average number of elements per cluster.

The variance of the above estimator (2.2) is given by

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$$V(\bar{y}_w) = \sum_{h=1}^{L} \lambda_h^2 \left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_{bh}^2 \tag{2.3}$$

where

$$S_{bh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(\frac{M_{hi}}{\bar{M}_h} \bar{Y}_{hi} - \bar{Y}_{h..}\right)^2$$

be the mean square between cluster means of h^{th} stratum.

$$\bar{Y}_{hi} = \frac{1}{M_{hi}} \sum_{j=1}^{M_{hi}} Y_{hij}$$

be the mean per element of i^{th} cluster in the h^{th} stratum.

3 Minimal equations and their approximate solutions for Proportional allocation

If clusters from each stratum are selected using proportional allocation, then we have

$$n_h = nW_h \tag{3.1}$$

Where

$$W_h = \frac{N_h}{N}$$

be the weight or proportion of clusters falling in the h^{th} stratum, and assuming that $M_{h0} \propto N_h$
and noting that $N_h = NW_h$
we may have $\lambda_h = W_h$ (3.2)

$$\text{and } \bar{M}_h = \frac{M_{h0}}{N_h} = \frac{M_0 W_h}{NW_h} = \frac{M_0}{N} = \bar{M}_0 \tag{3.3}$$

Now if finite population correction factor (f p c f) is ignored after using proportional allocation in each stratum and keeping in mind (3.2), we may have variance (2.3) as

$$V(\bar{y}_w)_p = \sum_{h=1}^{L} \frac{W_h S_{bh}^2}{n} \tag{3.4}$$

Now consider

$$S_{bh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(\frac{M_{hi}}{\bar{M}_h} \bar{Y}_{hi} - \bar{Y}_{h..}\right)^2$$

which can be rewritten on using (3.3) as

$$S_{bh}^2 = \frac{1}{\bar{M}_0^2} \cdot \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (T_{hi} - \bar{T}_{h..})^2 \tag{3.5}$$

where $T_{hi} = M_{hi} \bar{Y}_{hi}$ is the total of i^{th} cluster in the h^{th} stratum and

$$\bar{T}_{h..} = \frac{1}{N_h} \sum_{i=1}^{N_h} T_{hi}$$

is the mean of totals of clusters of the h^{th} stratum.
On using (3.5), we may rewrite the variance (3.4) as

$$V(\bar{y}_w)_p = \frac{1}{n\bar{M}_0^2} \sum_{h=1}^{L} W_h S_{bhT}^2 \tag{3.6}$$

Where

$$S_{bhT}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (T_{hi} - \bar{T}_{h..})^2$$

is the population mean square between totals of clusters of the h^{th} stratum. If N_h is large in every stratum, we may write variance (3.6) as

$$V(\bar{y}_w)_p = \frac{1}{n\bar{M}_0^2} \sum_{h=1}^{L} W_h \sigma_{bhT}^2 \tag{3.7}$$

Now it is clear from variance equation (3.7) that optimum points of stratification depend upon the distribution of totals of clusters. Therefore, we minimise the variance equation (3.7) with respect to T_h to get optimum points of stratification.

To proceed this, we lay down following assumptions:

1. Population π is infinite.
2. $f(T)$ exists and continuous.
3. The first two moments of $f(T)$ exist and are finite.

where $f(T)$ is the probability density function of the distribution of totals of clusters and is assumed to be known. Let $a < T_1 < T_2 \ldots . T_{L-1} < b$ be the points of stratification on the totals of clusters for character Y.

With the above assumptions, we may define following:

Let

$$W_h = \int_{T_{h-1}}^{T_h} f(T)\, dT \tag{3.8}$$

be the proportion of totals of clusters falling in the h^{th} stratum,

$$\mu_{hT} = \frac{1}{W_h} \int_{T_{h-1}}^{T_h} T f(T)\, dT \tag{3.9}$$

be the mean of totals of clusters for character Y in the h^{th} stratum, and

$$\sigma_{bhT}^2 = \frac{1}{W_h}\int_{T_{h-1}}^{T_h}(T - \mu_{hT})^2 f(T)\, dT \qquad (3.10)$$

be the variance between totals of clusters for character Y in the h^{th} stratum. Now for the minimisation of variance given in (3.7), we differentiate partially with respect to $T_h$and equating it to zero, we get

$$\frac{\partial}{\partial T_h}\left(\sum_{h=1}^{L} W_h\sigma_{bhT}^2\right) = \frac{\partial}{\partial T_h}(W_h\sigma_{bhT}^2) + \frac{\partial}{\partial T_h}(W_i\sigma_{biT}^2) = 0 \quad (3.11)$$

$h = 1,2 \ldots \ldots L-1;\ i = h+1$

Now using the definitions of W_h, μ_{hT} and σ_{bhT}^2 it can be verified that

$$\left.\begin{aligned}
&\frac{\partial}{\partial T_h}W_h = f(T_h)\\
&\frac{\partial}{\partial T_h}W_i = -f(T_h)\\
&\frac{\partial}{\partial T_h}(W_h\sigma_{bhT}^2) = (T_h - \mu_{hT})^2 f(T_h)\\
&\frac{\partial}{\partial T_h}(W_i\sigma_{biT}^2) = -(T_h - \mu_{iT})^2 f(T_h)\\
&\frac{\partial}{\partial T_h}\sigma_{bhT} = \frac{\left[(T_h - \mu_{hT})^2 - \sigma_{bhT}^2\right]}{2W_h\sigma_{bhT}} f(T_h)\\
&\frac{\partial}{\partial T_h}\sigma_{biT} = -\frac{\left[(T_h - \mu_{iT})^2 - \sigma_{biT}^2\right]}{2W_i\sigma_{biT}} f(T_h)
\end{aligned}\right\} \qquad (3.12)$$

Now substituting the values of partial derivatives from (3.12) in (3.11),we may get the minimal equations as

$$(T_h - \mu_{hT})^2 = (T_h - \mu_{iT})^2 \qquad (3.13)$$

$h = 1,2 \ldots \ldots L-1; i = h+1$

as $f(T_h) \neq 0$

These minimal equations (3.13) are difficult to be solved for their exact solutions as parameters in themselves depend upon the stratification points; therefore, we find the approximate solutions of these minimal equations. To find the approximate solutions of the minimal equations as obtained in (3.13) we shall obtain the series expansions of this system of equations about the point T_h, the common boundary of $h^{th}$and $(h+1)^{th}$ strata. Using Taylors theorem, the series expansions of μ_{hT} and σ_{bhT}^2 about the point $T = T_{h-1}$ which are given as

$$\mu_{hT} = \int_{T_{h-1}}^{T_h} Tf(T)dT \Big/ \int_{T_{h-1}}^{T_h} f(T)dT$$

$$= T_{h-1} + \frac{1}{2}k_h + \frac{f'}{12f}k_h^2 + \frac{ff'' - f'^2}{24f^2}k_h^3 + O(k_h^4) \qquad (3.14)$$

$$\sigma_{bhT}^2 = \frac{1}{\int_{T_{h-1}}^{T_h} f(T)dT}\cdot\int_{T_{h-1}}^{T_h} T^2 f(T)dT - \mu_{hT}^2 \simeq \frac{k_h^2}{12} \qquad (3.15)$$

where the function and its derivatives are evaluated at $T = T_{h-1}$ and $k_h = T_h - T_{h-1}$

Similarly expanding $\sqrt[\lambda]{f(t)}$ about the point $t = y$ from Taylor's theorem, it has been shown by Singh and Sukhatme (1969)

$$\left[\int_y^x \sqrt[\lambda]{f(t)}dt\right]^\lambda = k^\lambda\left[f(y) + \frac{f'(y)}{2}k + O(k^2)\right]$$

$$= k^{\lambda-1}\int_y^x f(t)dt\left[1 + O(k^2)\right] \qquad (3.16)$$

Using these above expansions, the system of minimal equations (3.13) reduces to

$$k_h^2\left[1 - \frac{f'}{3\,f}k_h + O(k_h^2)\right] = k_i^2\left[1 + \frac{f'}{3\,f}k_i + O(k_i^2)\right]$$

On raising both side to power 3/2 and using binomial theorem (for any index)

$$k_h^3\left[1 - \frac{f'}{3\,f}k_h + O(k_h^2)\right]^{3/2} = k_i^3\left[1 + \frac{f'}{3\,f}k_i + O(k_i^2)\right]^{3/2}$$

or

$$k_h^3\left[1 - \frac{f'}{2\,f}k_h + O(k_h^2)\right] = k_i^3\left[1 + \frac{f'}{2\,f}k_i + O(k_i^2)\right] \qquad (3.17)$$

On using the relation (3.16) with $\lambda = 3$, the system of equations (3.17) can be written in the form

$$k_h^2\int_{T_{h-1}}^{T_h} f(T)dT\left[1 + O(k_h^2)\right] = k_i^2\int_{T_h}^{T_{h+1}} f(T)dT\left[1 + O(k_i^2)\right] \quad (3.18)$$

If we have a large number of strata so that strata width k_h are small and their higher powers in the expansions can be neglected , then on neglecting the terms of order $O(m^4)$, where $m = \sup_{(a,b)}(k_h)$ on both sides of the system of equations (3.13) or equivalently (3.18), we have

$$\left[\int_{T_{h-1}}^{T_h} \sqrt[3]{f(T)}\,dT\right]^3 = \left[\int_{T_h}^{T_{h+1}} \sqrt[3]{f(T)}\,dT\right]^3$$

or

$$\int_{T_{h-1}}^{T_h} \sqrt[3]{f(T)}\,dT = \text{Constant } C$$

Where

$$C = \int_a^b \frac{\{f(T)\}^{1/3} dT}{L}$$

This gives us the following rule for finding the approximately optimum strata boundaries. **Cum $\sqrt[3]{f}$ rule:** If the function f(T)is bounded and possesses first two derivatives for all T in (a, b) then for a given value of L, taking equal intervals on Cum$\sqrt[3]{f}$ yields approximately optimum strata boundaries.

Table 1
Numerical Illustrations:

We consider three distributions for which optimum points of stratification have been obtained by approximate cum $\sqrt[3]{f}$ rule given by Ravindra Singh (1975) for proportional allocation.

1. Rectangular distribution
   $f(T)= 1;\ 1\leq f(T) \leq 2$
2. Right triangular distribution
   $f(T) = 2\ (2 - T);\quad 1\leq T \leq 2$
3. Exponential distribution
   $f(T) = e^{-T+1}\ ;\ 1\leq T \leq \infty$

Optimum Points of Stratification using Approximate Cum $\sqrt[3]{f}$ rule

f(T)	L	Approximate cum $\sqrt[3]{f}$ rule
Recatngular	2	1.5000
	3	1.3333, 1.667
	4	1.2500, 1.500, 1.7500
	5	1.2000, 1.400, 1.600, 1.800
Right Tringular	2	1.4060
	3	1.2627, 1.5623
	4	1.1944, 1.4060, 1.6474
	5	1.1543, 1.3188, 1.4977, 1.7020
Exponential	2	2.6262
	3	1.9806, 3.4461
	4	1.7048, 2.6262, 3.9594
	5	1.5505, 2.2221, 3.0911, 4.3164

4 Minimal Equations and Their Approximate Solutions for Equal Allocation

If clusters from each stratum are selected using equal allocation, then we have

$$n_h = \frac{n}{L} \tag{4.1}$$

The variance for Equal allocation is given by

$$V(\bar{y}_w)_e = \frac{L}{n\,\bar{M}_0^2}\sum_{h=1}^{L} W_h{}^2 \sigma_{bhT}^2 \tag{4.2}$$

Now minimising the of variance given in (4.2) with respect to T_h and equating to zero, we have

$$\frac{\partial}{\partial T_h}\left(\sum_{h=1}^{L} W_h^2\sigma_{bhT}^2\right) = \frac{\partial}{\partial T_h}\left(W_h^2\sigma_{bhT}^2\right) + \frac{\partial}{\partial T_h}\left(W_i^2\sigma_{biT}^2\right) = 0 \quad (4.3)$$

Now substituting the values of partial derivatives from (3.12) in (4.3), we get minimal equations as

$$W_h\left[(T_h - \mu_{hT})^2 + \sigma_{bhT}^2\right] = W_i\left[(T_h - \mu_{iT})^2 + \sigma_{biT}^2\right] \tag{4.4}$$

$h = 1,2, \dots\dots. L - 1;\ i = h + 1.$

as $f(T_h) \neq 0$

To find the approximate solutions of minimal equations using above expansions, the system of minimal equations (4.4) reduces to

$$k_h^3\left[1 - \frac{3f'}{4f}k_h + O(k_h^2)\right] = k_i^3\left[1 + \frac{3f'}{4f}k_i + O(k_i^2)\right]$$

On raising both side to power 2/3 and using binomial theorem (for any index)

$$k_h^2\left[1 - \frac{3f'}{4f}k_h + O(k_h^2)\right]^{2/3} = k_i^2\left[1 + \frac{3f'}{4f}k_i + O(k_i^2)\right]^{2/3}$$

or

$$k_h^2\left[1 - \frac{f'}{2f}k_h + O(k_h^2)\right] = k_i^2\left[1 + \frac{f'}{2f}k_i + O(k_i^2)\right] \tag{4.5}$$

On using the relation (3.16)with $\lambda = 2$, the system of equations (4.5)can be written in the form

$$k_h\int_{T_{h-1}}^{T_h} f(T)dT\left[1 + O(k_h^2)\right] = k_i\int_{T_h}^{T_{h+1}} f(T)dT\left[1 + O(k_i^2)\right] \quad (4.6)$$

If we have a large number of strata so that strata width k_h are small and their higher powers in the expansions can be neglected , then on neglecting the terms of order $O(m^3)$, where $m = \sup_{(a,b)}(k_h)$ both sides of the system of equations (4.4) or equivalently (4.6).

$$\left[\int_{T_{h-1}}^{T_h} \sqrt[2]{f(T)}\,dT\right]^2 = \left[\int_{T_h}^{T_{h+1}} \sqrt[2]{f(T)}\,dT\right]^2$$

$$\int_{T_{h-1}}^{T_h} \sqrt{f(T)}\,dT = \text{Constant } C$$

Where

$$C = \int_a^b \frac{\{f(T)\}^{1/2}\,dT}{L}$$

This gives us the following rule for finding the approximately optimum strata boundaries.

Cum $\sqrt{f}$ rule: If the function f(T) is bounded and possesses first two derivatives for all T in (a, b) then for a given value of L, taking equal intervals on Cum$\sqrt{f}$ yields approximately optimum strata boundaries.

5 MINIMAL EQUATIONS AND THEIR APPROXIMATE SOLUTIONS FOR NEYMAN ALLOCATION

If clusters from each stratum are selected using Neyman allocation, then we have

$$n_h = \frac{n W_h S_{bhT}}{\sum_{h=1}^{L} W_h S_{bhT}} \qquad (5.1)$$

The variance for Neyman allocation is given by

$$V(\bar{y}_w)_N = \frac{1}{n}\left(\sum_{h=1}^{L} W_h \sigma_{bhT}\right)^2 \qquad (5.2)$$

Now minimising the variance given in (5.2) with respect to T_h and equating to zero, we have

$$\frac{\partial}{\partial T_h}\left(\sum_{h=1}^{L} W_h \sigma_{bhT}\right) = \frac{\partial}{\partial T_h}(W_h \sigma_{bhT}) + \frac{\partial}{\partial T_h}(W_i \sigma_{biT}) = 0$$

$$\frac{\left[(T_h - \mu_{hT})^2 + \sigma_{bhT}^2\right]}{\sigma_{bhT}} = \frac{\left[(T_h - \mu_{iT})^2 + \sigma_{biT}^2\right]}{\sigma_{biT}} \qquad (5.3)$$

$h = 1, 2, \ldots \ldots \ldots L-1\,;\; i = h+1$

as $f(T_h) \neq 0$

To find the approximate solutions of minimal equations using above expansions, the system of minimal equations (5.3) reduces to

$$k_h\left(1 - \frac{f'}{4f}k_h + \frac{6ff'' - 5f'^2}{48 f^2}k_h^2 + O(k_h^3)\right) = k_i\left(1 + \frac{f'}{4f}k_i + \frac{6ff'' - 5f'^2}{48 f^2}k_i^2 + O(k_i^3)\right)$$

Taking squares both sides

$$k_h^2\left[1 - \frac{f'}{4f}k_h + O(k_h^2)\right]^2 = k_i^2\left[1 + \frac{f'}{4f}k_i + O(k_i^2)\right]^2$$

Or

$$k_h^2\left[1 - \frac{f'}{2f}k_h + O(k_h^2)\right] = k_i^2\left[1 + \frac{f'}{2f}k_i + O(k_i^2)\right] \qquad (5.4)$$

On using the relation (3.16) with $\lambda = 2$, the system of equations (5.4) can be written in the form

$$k_h \int_{T_{h-1}}^{T_h} f(T)dT\left[1 + O(k_h^2)\right] = k_i \int_{T_h}^{T_{h+1}} f(T)dT\left[1 + O(k_i^2)\right] \qquad (5.5)$$

If we have a large number of strata so that strata width k_h are small and their higher powers in the expansions can be neglected , then on neglecting the terms of order $O(m^3)$, where $m = \sup_{(a,b)}(k_h)$ both sides of the system of equations (5.3) or equivalently (5.5)

$$\left[\int_{T_{h-1}}^{T_h} \sqrt[2]{f(T)}\,dT\right]^2 = \left[\int_{T_h}^{T_{h+1}} \sqrt[2]{f(T)}\,dT\right]^2$$

$$\int_{T_{h-1}}^{T_h} \sqrt{f(T)}\,dT = \text{Constant } C$$

where

$$C = \int_a^b \frac{\{f(T)\}^{1/2}\,dT}{L}$$

This gives us the following rule for finding the approximately optimum strata boundaries.

Cum $\sqrt{f}$ rule: If the function f(T) is bounded and possesses first two derivatives for all T in(a,b) then for a given value of L, taking equal intervals on Cum$\sqrt{f}$ yields approximately optimum strata boundaries.

Table 2
Numerical Illustrations:

For illustration of cum$\sqrt{f(T)}$ rule we may consider a numerical example given in the book of Sampling Techniques by W.G. Cochran (1963, Third edition), On page 129 replacing $f(y)$ by $f(T)$ which is given below:
Calculation of Stratum Boundaries by the Cum $\sqrt{f(T)}$ rule

$\frac{\text{Industrial loan}}{\text{Total Loans}}\%$	f(T)	cum $\sqrt{f(T)}$	$\frac{\text{Industrial loan}}{\text{Total Loans}}\%$	f(T)	cum $\sqrt{f(T)}$
0-5	3464	58.9	50-55	126	340.3
5-10	2516	109.1	55-60	107	350.6
10-15	2157	155.5	60-65	82	359.7
15-20	1581	195.3	65-70	50	366.8
20-25	1142	229.1	70-75	39	373.0
25-30	746	256.4	75-80	25	378.0
30-35	512	279.0	80-85	16	382.0
35-40	376	298.4	85-90	19	386.4
40-45	265	314.7	90-95	2	387.82
45-50	207	329.1	95-100	3	389.5

6 CONCLUSION

In this paper, an attempt is made to obtain Approximately Optimum Strata Boundaries (AOSB) for cluster sampling design with unequal sizes of clusters using three different allocation methods. A Cum $\sqrt[3]{f}$ rule and Cum $\sqrt{f}$ rule of finding AOSB have been suggested for proportional allocation and equal, neyman allocation respectively. The determination of optimum strata boundaries in these cases have been illustrated with the help of some known specific distribution.

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