Optimization Of The Process Of Forming Pulse-Modulated Signals With Account For Hardware Restrictions

A.O. Ataullaev, O.Kh. Ataullaev, Kh.A. Salimjonov

Abstract: The article discusses the implementation of the content of setting objectives and given the decision to optimize formation process of radio pulse modulated signals with regard to the restrictions resulting from the proposed equipment telemetry systems (frequency bandwidth and peak power transmitting device). It is shown that proposed solution to the optimization problem is consistent with the data obtained by other authors in solving similar problems by other methods.

Index Terms: pulse-code modulation of signals, telemetry system, transmitting device, noise power, single-pole and three-pole filters and receiving devices.

1 INTRODUCTION

A sharp jump in recent years in the use of electronic systems in space and other fields of science and technology, with a simultaneous increase in the requirements for the dynamic and accuracy characteristics of such systems, has led to the widespread use of control theory methods in them. Increased requirements for electronic control systems make them use adequate mathematical apparatus. In this case, such an apparatus is the theory of dynamic optimization, which is widely used in the theory of optimal control. Let us turn to the problem of optimizing the formation of a pulse-modulated signal of a telemetry system taking into account hardware limitations (frequency bandwidth, peak power of a transmitting device) [1].

2 METHODOLOGY

The system (picture. 1) is described by equations with state variables of the nth order. In this case, the order of the equation is determined by the filter $\omega(t)$ of the transmitting device where a_i (for $i = 0,1,...,n-1$) are the filter coefficients, and x_1 is its output signal.

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_2, \\ &. \quad . \quad . \quad . \quad . \quad . \\ \dot{x}_n &= -a_0 x_1 - a_1 x_2, ..., -a_n x_n + a_0 m(t), \end{aligned} \quad (1)$$

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$$S_0(T) = \int_0^T u(t) x_1(t) dt.$$

The signal at the output of the receiving device at time T is determined by the ratio: (2)

$$\sigma_0^2(T) = N_0 \int_0^T u^2(t) dt,$$

The expression for the noise power at the output of the receiver is written as where N_0 is the noise power density, W/Hz.

(3)

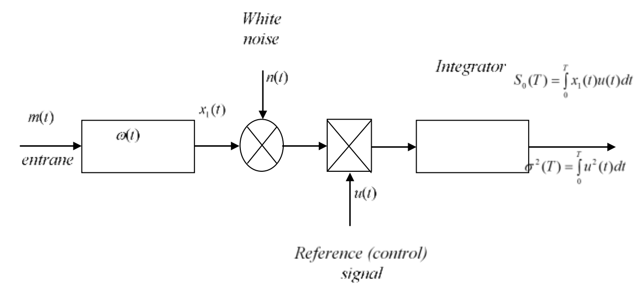

***Fig. 1**. Block diagram of the amplifier with auto-corrector.*

The energy and amplitude limits of the input signal (transmitting device) are respectively

$$E(T) = \int_0^T m^2(t) dt, \quad (4)$$

$$|m(t)| \le M.$$

(5)

The challenge is to maximize the signal/to-noise ratio of $S_0^2(T)/\sigma_0^2(T)$ under given constraints. This can be done with a fixed value of $S_0^2(T)$ by minimizing the noise output energy $\sigma_0^2(T)$. To go to the expanded phase space of the system, we introduce new state variables:

$$x_{n+1}(t) = \sigma_0^2(t) = N_0 \int_0^t u^2(t)dt,$$

$$x_{n+2}(t) = S_0(t) = \int_0^t u(t)x_1(t)dt,$$

$$x_{n+3}(t) = E(t) = \int_0^t m^2(t)dt. \qquad (6)$$

Differentiating them and combining them with (1), we obtain an extended system

$$\begin{aligned}
&\dot{x}_1 = x_2,\\
&\dot{x}_2 = x_3,\\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot\\
&\dot{x}_n = -a_0 x_1 - a_1 x_2, ..., -a_{n-1} x_n + a_0 m,\\
&\dot{x}_{n+1} = N_0 u^2,\\
&\dot{x}_{n+2} = u x_1,\\
&\dot{x}_{n+3} = m^2,
\end{aligned} \qquad (7)$$

or in vector form

$$\dot{x} = f(x, u, m). \qquad (8)$$

Now you need to minimize the variable $x_{n+1}(T)$ with the limited values of $x_{n+2}(T)$ and $x_{n+3}(T)$ the inequality $|m(t)| \le M$.

To solve this problem, the Pontryagin maximum principle is used. The Pontryagin function (in this case, the measure of the final error) is defined as the functional of states. It should be minimized or maximized at the final moment of time T by optimal control u(t). The function has the form

$$Q_1 = b \cdot x(T) = x_{n+1}(T) + b_{n+2} x_{n+2}(T) + b_{n+3} x_{n+3}(T). \qquad (9)$$

Hamilton function

$$H = H + \lambda^{\tau} f \qquad (10)$$

for the system in question is determined by the relation

$$H = \sum_{i=1}^{n+3} \lambda_i \dot{x}. \qquad (11)$$

Now with help

$$\lambda_i(t_1) = \frac{\partial \theta}{\partial x_i}$$

(12)

we find conjugate equations

$$\dot{\lambda}_i = -\frac{\partial H}{\partial x_i}, i = 1, 2, ..., n+3 \qquad (13)$$

with boundary conditions

$$\lambda(T) = [0, ..., -1 - b_{n+2} - b_{n+3}]^T. \qquad (14)$$

The minimum value of x_{n+1} with restrictions corresponds to the minimum value of the target functional Q_1, which is equivalent to the maximum value of H. In this case, the condition

$$H_{cu} = 0 \qquad (15)$$

$$\frac{\partial H}{\partial m} = \frac{\partial H}{\partial u} = 0. \qquad (16)$$

3 ANALYSES

Equation (16) determines the optimal functions m(t) and u(t), depending on λ_i and x_i. To find λ_i and x_i, it is necessary to solve a system of differential equations, including equations (7) and (13). Since the initial conditions x(0) are known for (7) and the final λ(T) for (13), in this case we have a two-point boundary problem, for the solution of which the trial and error method is used [1].

The process consists of estimating λ(0), followed by solving equations (7) and (13) and comparing the result with the values of λ(T) determined by (14). Approximation λ(0) continues until conditions (14) are satisfied. In [1], the calculation of a specific algorithm for the following numerical data is presented:

$$N_0 = 1{,}0\text{Вт/Гц},\ T = 1c,\ S_0(T) = 1\text{В},\ E(T) = 1\text{Вт}\cdot\text{c},$$
$M = 1{,}1$ and $M = \infty$

An initial value of $x_1(0) = -1B$ is adopted for the output of the transmitter $x_1(t)$. With these numerical values $x_1(0) = -1,$ $x_2(0) = x_3(0) = x_{n+3}(0) = 0.$ The problem is solved for two types of filters of the transmitting device: single-pole and three-pole (Chebyshev type). A single-pole filter is mainly used for comparison with the results of [2]. A three-pole filter is most typical for telemetry.

The optimal transmitter signal $m(t)$ and the reference (control) signal $u(t)$ for a single-pole filter are shown in picture. 2. The results obtained are in full agreement with the solution of a similar problem [2].

The transfer function $W(p)$ of a three-pole filter is described by the following relation:

$$W(p) = \frac{127{,}4}{p^3 + 6{,}4p^2 + 50p + 127{,}4}.$$

The above transfer function corresponds to a system of differential equations in variable states

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= -127{,}4x_1 - 50x_2 - 6{,}4x_3 + 127{,}4m, \\
\dot{x}_4 &= N_0 u^2 = u^2, \\
\dot{x}_5 &= x_1 u,
\end{aligned} \qquad (17)$$

with initial conditions

$$x_2(0) = x_3(0) = x_4(0) = x_5(0) = x_6(0) = 0.$$

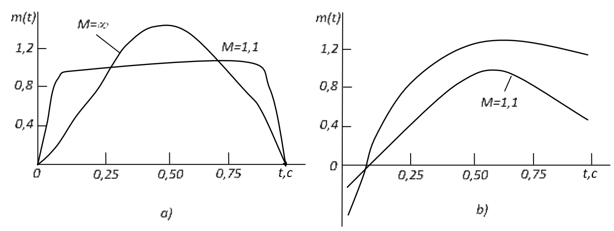

***Fig. 2**. Optimal (a) and reference (b) signals of a single-pole ...*

It is necessary to minimize 111, taking into account the limited values of 222 and 333 and the restrictions on the signal 444. The final error is written as

$$Q_1 = x_4(1) + b_5 x_5(1) + b_6 x_6(1). \qquad (18)$$

The Hamilton function for the problem in question has the form

$$\begin{aligned}
H &= \lambda_1\dot{x}_1 + \lambda_2\dot{x}_2 + \lambda_3\dot{x}_3 + \lambda_4\dot{x}_4 + \lambda_5\dot{x}_5 + \lambda_6\dot{x}_6 = \lambda_1 x_2 + \lambda_2 x_3 + \\
&+ \lambda_3(-127{,}4x_1 - 50x_2 - 6{,}4x_3 + 127{,}4m) + \lambda_4 u^2 + \lambda_5 x_1 u + \lambda_6 m^2.
\end{aligned} \qquad (19)$$

of

$$\lambda_i(t_1) = \frac{\partial\theta}{\partial x_i} \qquad (20)$$

we get the conjugate equations

$$\begin{aligned}
\dot{\lambda}_1 &= 127{,}4\lambda_3 - u\lambda_5, \\
\dot{\lambda}_2 &= 50\lambda_3 - \lambda_1, \\
\dot{\lambda}_3 &= 6{,}4\lambda_3 - \lambda_2, \\
\dot{\lambda}_4 &= \lambda_5 - \lambda_6 = 0.
\end{aligned} \qquad (21)$$

The boundary conditions for the system of equations (17) and (21) are written in the form

$$x(0) = [-100000]^T, \quad \lambda(1) = [0,-1,-b_5,-b_6]^T. \qquad (22)$$

according to

$$H_u = 0 \quad U = -U\,SgnB^T\lambda \qquad (23)$$

we get the equation

$$\frac{\partial H}{\partial u} = 0 = 2\lambda_4 u + \lambda_5 x_1.$$

or

$$m = -63{,}7(\lambda_3/\lambda_6), \lambda_6 \langle 0. \qquad (25)$$

The calculations were carried out for two values of M equal to 1.1 and ∞.

The system of equations (17) and (21) with boundary conditions (22) is a two-point boundary problem. The values of the coefficients b5 and b6 are found taking into account the fact that the limited values of $X_5(1)$ and $X_6(1)$ are equal to $X_5(1) = X_6(1) = 1$

To solve the system (17), (21) are given $\lambda_1(0)$, $\lambda_2(0)$ and $\lambda_3(0)$ so as to provide $X_5(1) = X_6(1)$. One way to find a solution is to bring the specially formed error function to zero. This function has the form

$$\pi = |x_5(1) - 1{,}0| + |x_6(1) - 1{,}0| + |\lambda_1(1)| + |\lambda_2(1)| + |\lambda_3(1)|. \qquad (26)$$

The initial values of λ(0) are systematically changed until the error becomes equal to zero. For a larger system, this procedure is greatly complicated.

In the case of telemetric pulse-code modulated (PCM) signals, an approximate solution can be used, taking it constant and equal to unity, thus eliminating one of the restrictions. The limitation remains only for the output signal

$$x_5(1) = S_0(1) = 1B$$

The system of equations of state variables will become shorter

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= -127{,}4x_1 - 50x_2 - 6{,}4x_3 + 127{,}4m, \\
\dot{x}_4 &= u_2, \\
\dot{x}_5 &= x_1 u
\end{aligned} \qquad (27)$$

with initial condition

$$x(0) = [-10000]^T$$

The conjugate system of equations can be written as

$$\begin{aligned}
\dot{\lambda}_1 &= 127{,}4\lambda_3 - \lambda_5 u, \\
\dot{\lambda}_2 &= 50\lambda_3 - \lambda_1, \\
\dot{\lambda}_3 &= 6{,}4\lambda_3 - \lambda_2, \\
\dot{\lambda}_4 &= \lambda_5 = 0
\end{aligned} \qquad (28)$$

with the final condition

$$\lambda(1) = [0,-1,-b_5]^T.$$

The equation for the support (control) function remains the same

$$u = -{}^1/_2(\lambda_5/\lambda_4)x_1. \qquad (29)$$

4 RESULTS AND CONCLUSION

Taking the value of the conjugate variable λ_4=-1, we select λ_5 so that the output signal $x_5(1)$ is equal to 1. This procedure uniquely defines u(t). Since the final data for the conjugate system are known, the initial data is found by solving the system in the inverse time. The results of the solution are shown in Fig. 3. The optimal waveform and reference (control) function for a three-link filter are shown in Fig. 4. Knowing the optimal signal, you can calculate the signal-to-noise ratio at the output of the receiving device.

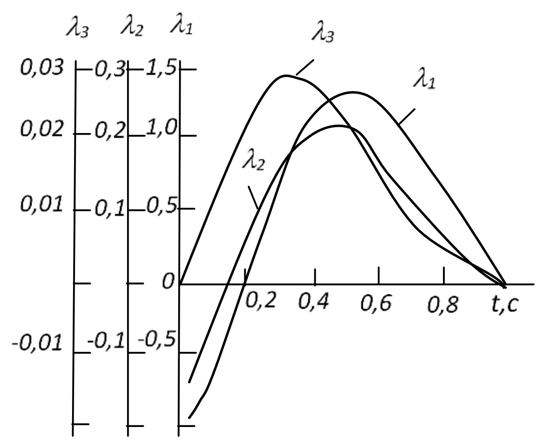

Fig. 3*. The optimal waveform of a three-link filter*

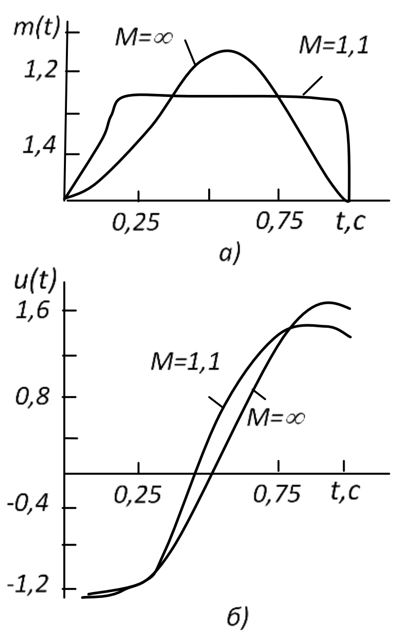

Fig. 4*. Support (control) functions m(t) and three-link filter u(t)*

When M=∞ it is equal - 0.84 dB.

To solve the two-point boundary problem, you can also use the methods developed [3-7]. Thus, the proposed solution to the optimization problem is consistent with the data obtained by other authors in solving similar problems.

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