Parametric Models Of The Third Degree For Output Parameters Of A CuBr Laser

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Abstract: Parametric third-degree models of laser output power and efficiency are developed for the first time on the basis of the large volume of experiment data on copper bromide vapor (CuBr) lasers. The results obtained using the regression models are compared against experimental data. Good correspondence is found with a relative error of 4-5%. Based on the obtained third degree regression equations, predictions are made for new laser sources with enhanced output characteristics - power and efficiency. The statistical adequacy and reliability of the results is assessed. The results are evaluated and interpreted.

Index Terms: Copper bromide vapor laser, laser efficiency, laser output power, parametric model, polynomial model, prediction, regression analysis, regression model.

1 Introduction

IT is accepted that metal vapor and metal compound vapor lasers have been studied in great detail through experiments and in theory. During the last few years as a result of active experiment development, a significant increase (up to 120-150 W) of the output power was achieved. This substantially expanded the scope of applications. Copper bromide (CuBr) vapor lasers continue to be the most powerful laser source in the visible spectrum (510,6 nm and 578,2 nm). This is the reason for its active application in medicine, physics, chemistry, ecology, technology, and scientific research. There are over 2000 scientific publications, papers, books, and monographs in renowned science journals in this field, as well as papers presented at prestigious science conferences. Copper bromide vapor lasers are a modified version of pure copper vapor lasers. This paper examines variations of this laser, invented and developed at the Laboratory of Metal Vapor Lasers of the Georgi Nadjakov Institute of Solid State Physics at the Bulgarian Academy of Sciences, Sofia. The first patent related to this laser is that of N. V. Sabotinov, et al., Bulg. patent No.:28674, 1975. The copper bromide vapor laser is one of the 12 laser sources which have a wide range of applications and are commercially viable [1, 2]. Over the past 30 years had accumulated a large volume of experimental material that is the subject of active survey [2-4]. With the help of statistical products such as SPSS, MARS, CART, a detailed classification analysis of dependent and independent parameters of the CuBr laser were developed, as well as parametric and nonparametric models for laser output power and laser efficiency.

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The purpose of these statistical models was to explore the complex nature of the processes and its relations in the active laser volume to intensify technical and engineering improvement of existing laser sources and develop new ones. Nonparametric models for laser power and efficiency for CuBr laser emitting in the visible range are developed in [3, 8]. Based on MARS software product and method of multidimensional adaptive regression splines, models of up to the thrid degrees were derived. This allowed to obtain a good fitting with the experimental results. However, the application of MARS models have some disadvantages. In particular, they are difficult to be interpreted directly. Also, these results should be able to confirm with other methods.The aim of this study is to obtain a parametric model of the third degree. In this model explicitly must be set dependence of the laser output and efficiency on the laser operational characteristics (independent variables). This would significantly broaden its application. The obtained parametric equations could be directly used by means of widely known software products such as Excel, Matlab, Mathematica. This article is a continuation of [2, 5-7]. In these articles were developed parametric models of first and second degree for prediction of laser power and efficiency. The purpose of this article is to continue research on parametric statistical models by developing models of the third degree. This will allow us to obtain regression equations of the third degree, which more adequately describe the complex non-linear processes in the laser tube. They can more accurately predict the laser power and efficiency than previously developed in [2, 5-7] parametric models of first and second degree. For the basis of these models will be used some earlier results obtained with nonparametric MARS methods [8].

2 Object of Study

Copper bromide vapor lasers are well-known as sources of pulse radiation in the visible spectrum (400-720 nm) emitting at two wavelengths: green 510.6 nm and yellow - 578.2 nm. They are considered to be high-pulse lasers. Neon is used as a buffer gas. In order to improve efficiency, small quantities of hydrogen are added. Unlike the high-temperature pure copper vapor laser, the copper bromide vapor laser is a low-temperature one, with an active zone temperature of 700°C. The laser tube is made out of quartz glass without high-temperature ceramics as a result of which it is significantly cheaper and easier to manufacture. The discharge is heated by electric current (self-heating). It produces light impulses tens of nanoseconds long. Its main advantages are: short initial heating period, stable laser generation, relatively long service life, high values of output power and laser efficiency.

Fig. 1 shows a schematic of the laser tube.

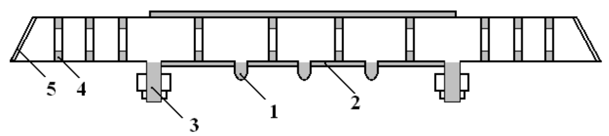

Fig. 1. Construction scheme of CuBr laser tube: 1- reservoirs with copper bromide, 2- heat insulation of the active volume, 3- copper electrodes, 4- inner diaphragms, 5- tube windows.

We will investigate the following 10 laser characteristics (independent variables, predictors): *D*, mm - inside diameter of the laser tube; *dr*, mm - inside diameter of the rings; *L*, cm - distance between the electrodes; *Pin*, kW - supplied electric power in the discharge; *PH*2, Torr - hydrogen pressure; *PL*, kW/cm - input power per unit length; *PRF*, kHz - pulse repetition frequency; *Pne*, Torr - neon gas pressure; *C*, nF - equivalent capacity of the condenser battery; *Tr*, °C - temperature of the CuBr reservoirs. As dependent output characteristics, will be considered: *Pout*, W - output laser power (laser generation) and *Eff*, % - laser efficiency.

3 Reduced Third Degree Model of Laser Output Power (*Pout*)

The best nonparametric third order model for *Pout* generated by MARS [8] contains a total of 45 basis functions of first, second, and third order. This required the participation of just 8 independent variables: *Pin*, *C*, *PRF*, *PL*, *dr*, *PH*2, *Tr* and *Pne*. The variables *D* and *L* have been removed from the model as non-significant. These 45 basis functions contain only three variables in the third degree terms: {*PL*, *C*, *Pin*}, {*PRF*, *Tr*, *PH*2} and {*PRF*, *C*, *Pin*}. Since the MARS model excludes repetitions, in our modeling procedure we add all 10 independent variables in the third degree terms. The third order regression equation for Pout is sought in the form:

$$\begin{aligned}\hat{P}out &= a_0 + \sum_{i=0}^{10}\sum_{j=i+1}^{10} a_{ij} y_{ij} y_{ij} + a_{66}.PL.C.Pin \\ &+ a_{67}.PRF.Tr.PH2 + a_{68}.PRF.C.Pin + a_{69}.D^3 \\ &+ a_{70}.L^3 + a_{71}.dr^3 + a_{72}.Pin^3 + a_{73}.PL^3 + a_{74}.PRF^3 \\ &+ a_{75}.Pne^3 + a_{76}.PH2^3 + a_{77}.C^3 + a_{78}.Tr^3\end{aligned} \quad (1)$$

The first two terms in (1) represent all possible combinations of first and second degree, including repetitions of the 10 independent variables, a total of 66 unknown coefficients and allows short recording of equation (1). The model of the third degree is reduced because from formally considered and included all possible combinations of 10 independent variables of the third degree, only 13 of them are taken, in accordance with the results of MARS model. To build the model we used the statistical package SPSS and carried out the regression procedure with a Backward mode. Statistically significant final model was reached after 21 steps. The resulting non-standardized regression equation for *Pout* is as follows:

$$\begin{aligned}PoutPre &= 10,243 + 64,453.Pin - 0,143.Tr \\ &+ 0,023.D.PRF + 0,303.D.C - 0,061.dr.PL \\ &- 0,011.L.PRF - 0,419.L.C - 2,009.Pin.PL \\ &+ 0,437.Pin.PRF + 2,341.PL.PH2 - 2,791.PL.C \\ &- 0,579.PRF.C + 0,165.C.Tr - 0,011.D^2 \\ &+ 0,025.dr^2 - 41,015.PH2^2 - 11,333.C^2 \\ &+ 0,003.PL^3 - 2,1.10^{-5}.PRF^3 + 1.613.C^3\end{aligned} \quad (2)$$

It has to be noted that equation (2) contains only three third degree terms: PL^3, PRF^3 and C^3.

TABLE 1

COMPARISON OF THE HIGHEST EXPERIMENTAL VALUES OF LASER OUTPUT POWER POUT WITH THE FITTED MODEL VALUES, OBTAINED BY EQUATION (2).

Pout, W	*PoutPre*, W	Relative Error δ, %
108	111.3	3.09
110	105.3	4.28
112	105.6	5.70
112	105.4	5.89
112	105.8	5.51
118	111.8	5.27
120	112.0	6.67
120	112.9	5.95
120	112.4	6.31

Table 1 compares the highest experiment results of *Pout* and those obtained using model equation (2) (*PoutPre*). The last column shows the relative error for each row. The mean relative error in the region with the highest laser output is 5.41%. This is the best result out of all parametric models of Pout of the first and second degree [2, 5-7]. The obtained third degree regression model (2) is also the best when considering its statistical indices. E.g. R=0.998; R^2=0.976, Std. Error of the Estimate = 5. 677. In Table 2, the behavior of laser output power *Pout* is predicted based on equation (2) for out-of-data sample. For the sake of simplicity, the geometric design parameters (*D*, *dr*, *L*) have not been changed. For comparison, the first row shows the experiment with the optimal value of the laser output power *Pout* (120W). The table indicates that in order to increase *Pout*, the quantities *Pin*, *PRF* and *Tr* need to be increased, and *PH*2 and *C* - decreased. Although model (2) is a third degree one, as shown by equation (2) and Table 2, the increase of laser output power is most strongly influenced by the quantity of *Pin*, as confirmed by the results obtained so far by the models of the first and second degree in [2, 5-7]. It was obtained that the standardized residuals for equation (2) are normally distributed, which indicates the adequacy of the developed model.

TABLE 2
PREDICTED VALUES FOR A "HYPOTHETICAL EXPERIMENT" UNDER REGRESSION MODEL (2)A.

Pin, kW	*PL*, W/cm	*PH*2, Torr	*PRF*, kHz	*Pne*, Torr	*C*, nF	*Tr*, °C	*PoutPre*, W
5.00	12.50	0.60	17.5	20	1.30	490	120.0
5.10	12.75	0.55	18.0	0.55	1.29	495	123.2
5.15	12.875	0.50	18.5	0.50	1.28	495	125.2
5.20	13.00	0.40	19.0	0.40	1.25	500	128.7
5.50	13.75	0.35	20.0	0.35	1.22	505	133.8
5.60	14.00	0.30	21.0	0.30	1.20	510	136.5

A comparison between experiment data and the respective fitted values calculated using model equation (2) is shown in Fig. 2. It is apparent that the confidence interval for 95% contains 97.6% of the experiment data. This is the highest percentage when compared to all parametric polynomial models in [2, 5-7]. This once again indicates that model (2) provides the best prediction of laser output power *Pout*.

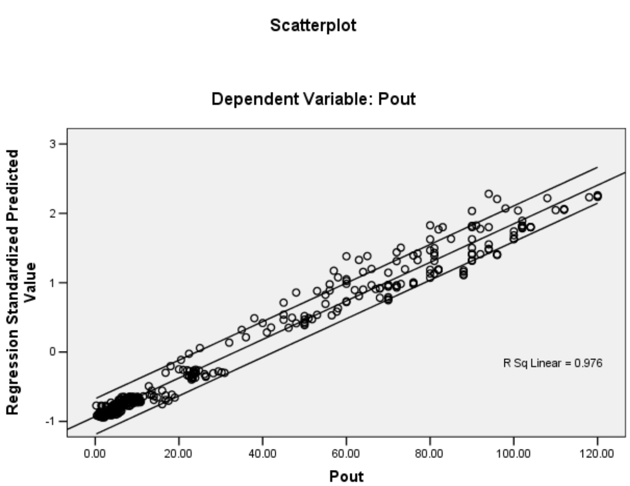

Fig. 2. Comparison of the experimental results of laser output power *Pout* against the regression standardized predicted values.

4 Reduced Third Degree Model of Laser Efficiency *Eff*

By analogy to above section 3, we take into account the fact, that the best nonlinear third order MARS model of *Eff* contains 44 basis functions of first, second, and third degree terms. They include 9 out of a total of 10 independent variables, namely: *D*, *dr*, *Pin*, *PL*, *PRF*, *PH*2, *Pne*, *C* and *Tr*. Only the quantity *L* is excluded. Its nonlinear third order elements participated in 7 terms are: {*C*, *PRF*, *Pin*}, {*Pne*, *PRF*, *Pin*}, {*Pin*, *C*, *dr*}, {*PL*, *PRF*, *PH*2}, {*D*, *PRF*, *D*}, {*C*, *PL*, *PRF*} and {*D*, *PRF*, *PH*2}. To these, once again we add each of the 10 independent variables to the power of three. In this manner, the nonlinear equation takes the following general form:

$$\begin{aligned}\hat{Eff} &= a_0 + \sum_{i=0}^{10}\sum_{j=i+1}^{10} a_{ij} y_{ij} y_{ij} + a_{66}.C.PRF.Pin \\ &+ a_{67}.Pne.PRF.Pin + a_{68}.Pin.C.dr + a_{69}.PL.PRF.PH2 \\ &+ a_{70}.D.PRF.D + a_{71}.C.PL.PRF + a_{72}.D.PRF.PH2 \\ &+ a_{73}.D^3 + a_{74}.L^3 + a_{75}.dr^3 + a_{76}.Pin^3 + a_{77}.PL^3 \\ &+ a_{78}.PRF^3 + a_{79}.Pne^3 + a_{80}.PH2^3 + a_{81}.C^3 + a_{82}.Tr^3\end{aligned} \quad (3)$$

Stepwise regression was applied to determine the statistically significant coefficients. The resulting third degree regression equation for laser efficiency *Eff* is the following:

$$\begin{aligned}EffPred &= 0,934 + 0,075dr.PH2 - 0,005Pin.C.dr \\ &- 3.122PH2^3 + 1.518.Pin - 0,018.PH2.Pne \\ &- 0,001.D.PL + 0,005.C.PL.PRF - 9,9.10^{-5}.PRF^2 \\ &- 0,064.Pin^2 - 0,049.PRF.C - 0.128.PL \\ &+ 0,129.PL.PH2 - 0,966.Pin.PH2\end{aligned} \quad (4)$$

The adequacy of the developed regression model has been checked. In order to do this, equation (3) is used to calculate the laser efficiency for known experiment results. Some of the results for the highest values of output power are given in Table 3. The first column shows experiment values for laser output power *Pout*. The second - the respective values of supplied electric power *Pin*. And the third column contains the values of laser efficiency calculated using the formula *Eff*=100 *Pout*/*Pin*, %. Column 4 shows predicted results for laser efficiency in accordance with equation (4), and the last column 5 - the relative error in percent.

TABLE 3
COMPARISON OF LASER EFFICIENCY BETWEEN EXPERIMENT DATA AND PREDICTED VALUES USING EQUATION (4).

	Pout, W	*Pin*, kW	*Eff*, %	*EffPred*, %	δ, %
	1	**2**	**3**	**4**	**5**
1	101	4.5	2.24	2.37	5.40
2	102	5.0	2.04	1.95	4.51
3	102	3.5	2.91	2.76	5.16
4	104	4.0	2.60	2.41	7.29
5	108	5.0	2.16	2.28	5.75
6	110	4.5	2.44	2.37	2.93
7	112	4.5	2.49	2.38	4.24
8	112	4.5	2.49	2.39	3.97
9	118	5.0	2.36	2.30	2.55
10	120	5.0	2.40	2.31	3.86
11	120	5.0	2.40	2.34	2.60
12	120	5.0	2.40	2.32	3.22

The mean relative error is 4.29%. The statistical indices are as follows: R=0.974, R^2=0.949, Std. Error of the Estimate = 0.205. When compared to the second degree model of *Eff* [7], the developed model (4) is better. It predicts more precisely known experiment results and has better statistical indices. The next step is to predict the new behavior of laser efficiency for data other than those from experiments using equation (4). Some of the results are given in Table 4. The geometric dimensions *D*, *dr* and *L* are fixed. The computer simulations indicate that in order for *Eff* to increase, the quantities *Pin*, *PL*, *PH*2, *Pne* and *C* need to be decreased. The results are partially confirmed by previously obtained data as given in [7] for the second degree models.

TABLE 4
PREDICTION OF LASER EFFICIENCY FOR NEW LASER SOURCESA.

Pin, kW	*PL*, W/cm	*PH*2, Torr	*PRF*, kHz	*Pne*, Torr	*C*, nF	*Tr*, ^{o}C	*EffPred*, %
5.0	12.5	0.6	17.5	20	1.30	490	2.40
4.9	12.25	0.6	17.5	20	1.30	490	2.41
4.9	12.25	0.5	17.5	19	1.30	490	2.61
4.8	12.00	0.5	18	19	1.25	490	2.66
4.8	12.00	0.45	18	18	1.25	490	2.73
4.7	11.75	0.45	19	18	1.20	490	2.79
4.7	11.75	0.4	19	18	1.20	490	2.82

[A] The values of geometric parameters are fixed as *D*=*dr*=58 mm, *L*=200cm.

5 Discussion with Conclusion

The developed third degree models of laser power and efficiency show that the processes which determine their behavior are strongly nonlinear. This is a significant obstacle when performing detailed analysis of the influence of each of the 10 independent quantities. In practice, such an analysis is impossible. We can only seek qualitative confirmation by means of previous conclusions from simpler models. The developed nonlinear models of third degree are better than all other lower order ones. The investigations with nonlinear parametric equations using fourth and higher order factors [2] indicate that their statistical indices are worse. Therefore, nonlinear third order parametric models provide the most accurate description of the processes within the active laser tube. The analysis of the computer simulations, Tables 2 and 4, shows that laser sources with enhanced output parameters may be constructed without altering the geometric dimensions of the laser tube. This conclusion enlarges the results of linear model 1 in [2], where an increase in *Pout* required larger geometric dimensions. Nonlinear solutions, where only some independent quantities are modified, provide opportunities for the development of the considered lasers. Increasing laser output power and efficiency while maintaining the geometric design allows for numerous structural components of the laser system to be kept: laser resonator, laser support structure housing the laser tube, the laser system developed for industrial or technological application of the laser source. Practically, the old laser tube simply needs to be replaced by a new one with higher output characteristics but the same geometric dimensions. This possibility defines the most crucial conclusion for the developed nonlinear models.

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