

# Formation Control For Wheeled Mobile Robots

Ritesh Meshram, Dr. Sunil Surve

**Abstract:** this paper describes work on formation for a group of wheeled mobile robots. Also in this paper we have tried to give the detail information about the kinematics of wheeled mobile robot available in various literatures in a simpler way. We are using car-like mobile robot where the leader mobile robot is controlled to reach the desired position, whereas the follower follows the leader robot with some specific distance and angle. Here we made an assumption that the motion occurs in a 2D space without any obstacle in its path. The basic of kinematic bicycle model for a wheeled mobile robot is used in this work. For controlling the motion of robots we are using fuzzy logic. As we know, Fuzzy logic theory is subdivided into two types, type-1 and type-2. For the motion control of a car-like mobile robot in formation we are using the concept of Interval type2 fuzzy logic (IT2-FL). A new rule base is designed for IT2-FL system and the simulation is done in Matlab environment also the final results are given to show the efficiency and effectiveness of the proposed fuzzy controller.

**Index Terms:** car-like, mobile robot, IT2-FL, kinematics, leader-follower.

## 1 INTRODUCTION

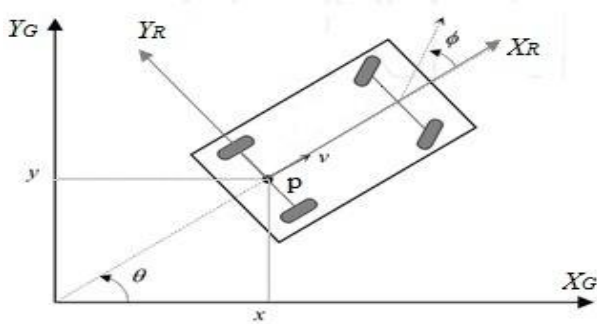
In the last 3 decades, the idea of robots doing human works has come to reality which was earlier considered as cannot be done and just an imagination. Robotics devices and machineries are now becoming the part of human life. Now a days robots are very commonly used in service industry, armed forces, manufacture industry, etc. The one of the most challenging and demanding field of robotics is wheeled mobile robots. One of the reasons for this is mobile robot needs mobility that helps it to move unrestrained all over its environment. As given in [1], a mobile robot can move its surrounding in many different ways. Therefore for a mobile robot, how it moves in its environment depends upon the type of approach used for its motion. A substantial portion of wheeled mobile robotics exploration/research includes developing robot model that mimic a car-like motion without the help of any human being. Also it has been seen as in case of well-known work in the field of mobile robotics the system strategy need not required to be very complex even with simpler one we can achieve our desired result. In case of mobile robotics, sometimes it becomes necessary that to perform a specific job we need group of mobile robot in formation. The requirement for formation of a group of mobile robot performing a specific task has leads to the growth of a new research field. The formation control problem for mobile robots can be defined as finding a system which makes sure that the group of mobile robots can hold on a given formation or precise set of formations. The objective of this work is to develop leader-follower structure, such that multiple mobile robots can move in a given condition in formation. We are considering an obstacle free environment for our work. At first we need to know how our of mobile robot move in its surroundings. For understanding this process we need to understanding kinematic equation of motion. Kinematics is said to be a geometrical or mathematical analysis of how our system behaves in its environment without taking into considering the forces that cause this.

For understanding the Kinematics of car like mobile robot first we must know the type of wheels we are using in our case and also the constrained applied due to it. In common case disk type wheels are used for a carlike wheeled mobile robot. In case of simulation, a car like model can be designed using simple geometric and trigonometric concept for more understanding please refer [2]. Many control approaches have been put forward to solving the motion control problem such game theory, model predictive control and so on. We have use Fuzzy logic (type-2) in our case. Seong-Gon Kong in [4] presented a work in type1 fuzzy logic controller having two input and one output state variable. x coordinate and vehicle orientation where the input to the controller whereas the output is the vehicle steering angle. The rule base used is a very common rule base used for mobile robot case, nearly similar kind of rule base is given in [5]-[7]. We have designed a new rule base having 25 rules for our case. Many times it is seen that the data that is used to develop the rules for a fuzzy system are uncertain. Due to the uncertainty, like an uncertainty in collection of information for a same case by different people, it becomes tough to decide what the exact value of membership function is. The overall structure of fuzzy sets in a fuzzy system is to allow the handling and modeling of much of uncertainty using type-1 fuzzy sets. But Type-1 fuzzy logic systems are not able to completely handle the linguistic or numerical uncertainties [9]. In such a case we need another kind of fuzzy set for a fuzzy system called as type 2 fuzzy set. A type-2 fuzzy logic system or controller uses notation that are very much similar to that are used in a type-1 fuzzy logic controller such as membership functions, fuzzy rules, t-norms operations, fuzzification, inference, defuzzification. The only difference is that addition of type reduction process before defuzzification. The basic ideas of IT2-FL in Matlab is explained by Dongrui Wu in [8] whereas Hani hagrass [9] explains the theoretical concept of type2 FLS in simple language. The paper is arranged as follow Section II gives kinematic information of mobile robot along with kinematic constraints, also telling why we are having these constraint and how our mobile robot move in its environment, section III is focused on our leader-follower formation problem. Section IV gives the theoretical concept of Type2 fuzzy logic along with the proposed controller. Section V shows the simulation results whereas Section VI tells the conclusion.

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## 2 WHEELED MOBILE ROBOT

The wheeled mobile robot can be of different type, but as said earlier we are using car-like model. Consider the car-like robot shown in in figure.



**Fig.1:** The global reference frame and the robot local reference frame.

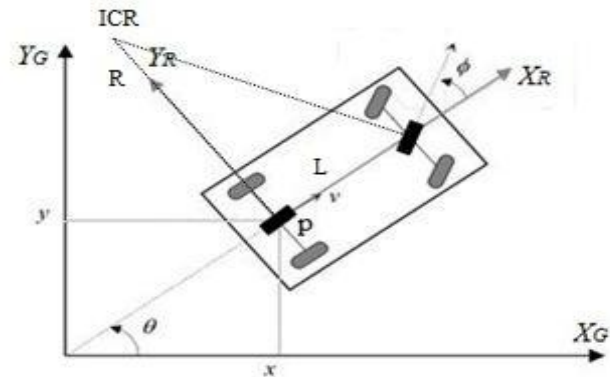
The first thing we need to know is Frames. Every object or as in our case mobile robot has its own frame, generally called as local body frame. Not only this the environment in which the mobile robot is moving or doing his work is also considered as frame, called as global or world frame, as you can see in figure 1, the world frame is represented by axes  $(X_G, Y_G)$  whereas robot local body frame is represented by  $(X_R, Y_R)$  axes. For more information on frames please refer [11]. Let point p be considered as the origin of the body of mobile robot. The heading direction or longitudinal direction of the robot local body frame is represented by axis  $X_R$  whereas the lateral direction of mobile robot local body frame is represented by the axis  $Y_R$ . The configuration space of a mobile robot at any given time can be defined by a set of four state variables. Basically a configuration space is a set of all variable which states what a system is doing at a given time. In case of configuration scape of car-like mobile robot,  $(x, y)$  represent the position of the origin point p of the mobile robot local body frame with respect to world reference frame,  $\theta$  represent the orientation angle of mobile robot whereas the steering angle is represented by  $\phi$ , it is the angle between mobile robot's front wheel and the mobile robot's longitudinal axis  $X_R$ . The configuration space of mobile robots is given as;

$$q = \begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix} \quad (1)$$

### 2.1 Kinematic Bicycle Model

As given in [1], during the course of the study we are considering the mobile robot as a stiff body to which wheels are attached, operating on a horizontal plane. Using rotation matrix we can map the position of mobile robot it term of

global reference frame, for more information on rotation matrix please refer [10].



**Fig. 2:** Theoretical scheme of bicycle/car

Using bicycle model we can streamline the Ackerman steered model for a wheeled mobile robot. We can have bicycle model by combining the left and right wheels into a pair of single wheels at the center of the front and rear axles as shown in dark black color in Figure 2. The equation of motion for the kinematic bicycle model of a car can be understood from [3]. Before going to kinematic equation of motion one important notation we must know that is ICR (instantaneous center of rotation). We know that the regular wheel that is disk type wheels have sliding restrictions. Therefore they have zero lateral motion. As shown in figure 2, to show this geometrically we have to draw a line called as zero motion line perpendicular to the longitudinal axis of wheels shown in dotted lines. The intersection point of the zero motion line of all wheels gives a single point called ICR. Due to ICR when a robot takes turn, the wheel must be rolling along a circle such that the center this circle is our point ICR and its radius is represented by R. The radius ICR will be infinite when mobile robot is moving in straight line. Therefore a mobile robot like an Ackerman vehicle as shown Figure 2 may have a number of wheels, but it is essential it have a single ICR as in case of bicycle. This ICR geometric construction illustrates how robot mobility depends on the number of constraints on the robot's motion but not the number of wheels [1], [18].

### 2.2 Kinematic Equation of Motion

As we said earlier we are considering car as a rigid body moving on the plane. The configuration or posture of the vehicle can be described with three coordinates in the world frame  $q = (x, y, \theta)$ . Differentiating with respect to time we can write the configuration of mobile robot in terms of velocity  $\dot{q} = (\dot{x}, \dot{y}, \dot{\theta})$ . L is the distance between the front and rear wheels. If the steering angle  $\phi$  is set at some fixed value, the car will travels in a circular motion and the radius of the circular motion is R. let the speed of robot be  $V_x$  in  $X_R$  direction and  $V_y$  in  $Y_R$  direction. The speed of the vehicle in  $Y_R$  direction is zero because the wheels cannot slip sideways. Therefore we have  $V_x = V$  and  $V_y = 0$ . Thus we can represent the motion of vehicle using two dimensional rotation matrix as [10][11]

$$\begin{aligned}\dot{x} &= v \sin \theta \\ \dot{y} &= v \cos \theta\end{aligned}\quad (3)$$

$\dot{x}$  and  $\dot{y}$  are vector components of linear velocity  $v$  and  $\theta$  is the orientation angle of mobile robot local body frame w.r.t world frame. The next task is to derive the equation for angular velocity. The angular velocity is given as

$$\dot{\theta} = \frac{v}{R}$$

From trigonometry  $R = \frac{L}{\tan \phi}$ , refer figure 2.

Thus the angular velocity of mobile robot becomes

$$\dot{\theta} = \frac{v \tan \phi}{L}\quad (4)$$

### 2.3 Kinematic Constraints

These constraints are also referred as nonholonomic constraints where the system has restrictions in its velocity, but the important point to be noted that these restrictions do not cause any restrictions in its positioning. From a kinematic point of view, the main characteristic of wheeled robots is the nonholonomic rolling without slipping constraint of the wheels on the floor, which forces the vehicle to move tangentially to its main axis. This reduces the set of accessible velocities at each time makes the path planning problem particularly difficult. Viewed another way, the system's local movement is restricted, but not its global movement. Mathematically, this means that the velocity constraints cannot be integrated to give position constraints. The nonholonomic constraints can be derived from equation 3 as;

$$\dot{x} \sin \theta - \dot{y} \cos \theta = 0\quad (5)$$

The constraint of "rolling without slipping" means that and must be consistent with the direction of rolling, they are not independent [20]. This means that the robot cannot change position and direction independently.

### 3 LEADER-FOLLOWER FORMATION

Leader follower strategy can be of various types like there can many leaders and many followers or one leader and many followers and so on. In our case we have use one leader and much follower strategy. For simplification we have assumed that the follower already have information where the robot is. The equation for our formation framework is as follows

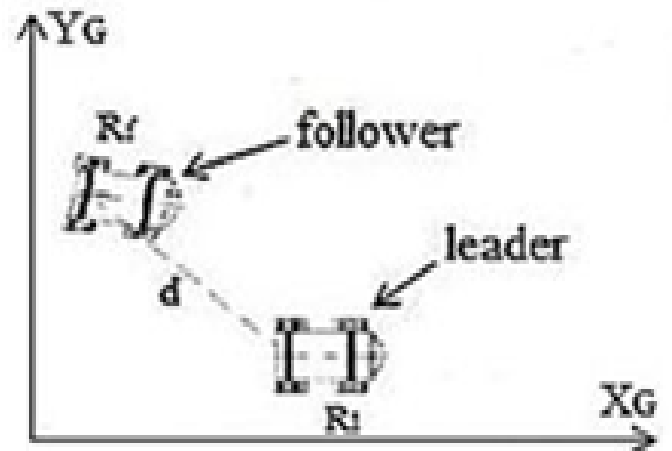


Fig. 3: Leader-Follower Formation strategy

$$x_f = x_l + d$$

$$y_f = y_l + d$$

$$\theta_f = \theta_l$$

Where  $(x_f, y_f)$  and  $(x_l, y_l)$  are position of follower and leader mobile robot respectively whereas  $\theta_f$  and  $\theta_l$  are the orientation of follower and leader robots respectively.  $d$  is the desired distance between leader and followers.

### 4 TYPE 2 FUZZY LOGIC

Type1 fuzzy logic has been used from around 1965. It was introduced by Zadeh. Ten years later he introduces type 2 fuzzy logic which is considered as an extension of type 1 fuzzy logic. Type1 fuzzy logic from the time of its introduction has been used in number of applications. But there is a problem in type 1 fuzzy logic, the membership grade or the degree of truth for each input is a crisp value ranging from 0 to 1 refer [13][19]. In many cases this works quite well and produces an output for a system such that it can do its job effectively. Fuzzy logic systems are widely used due to their ability to model and handle uncertainty. But some time it is not possible using type 1 fuzzy logic system. Thus in this case type two fuzzy logic is used. Type-2 fuzzy sets increased scope for modeling uncertainty better. Uncertainty can be of different type e.g. In fuzzy logic we deal with words, but words mean different to different people [12][14]-[16][19]. For understanding the concept of Interval type2 fuzzy logic please refer [13]-[15]. A type 2 fuzzy set is described by footprint of uncertainty and secondary membership function. A type 2 fuzzy set is said to be bounded by two type1 fuzzy set. A type-2 fuzzy set is bounded from below by a Lower Membership Function (LMF) from above by an Upper Membership Function (UMF). The area between LMF and UMF is called footprint of uncertainty (FOU).

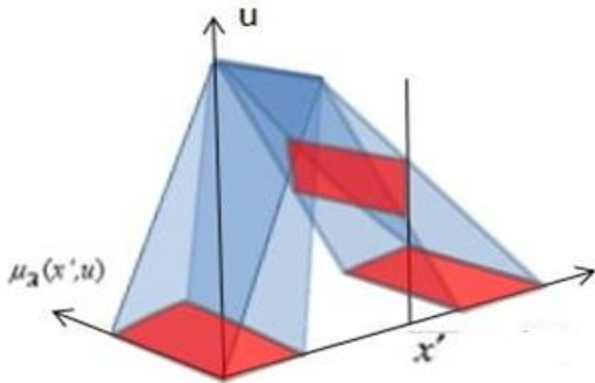


Fig. 4: 3D view of interval type-2 fuzzy set

In fig.4,  $x'$  is input,  $u$  is primary membership function,  $\mu_A(x',u)$  is secondary membership function and the area shaded in red color is represented by FOU. It is the 3<sup>rd</sup> dimension of secondary membership function and the FOU area which maps the uncertainty of primary membership in secondary membership function and helps us to handle and model uncertainty in better way than type 1 fuzzy set.

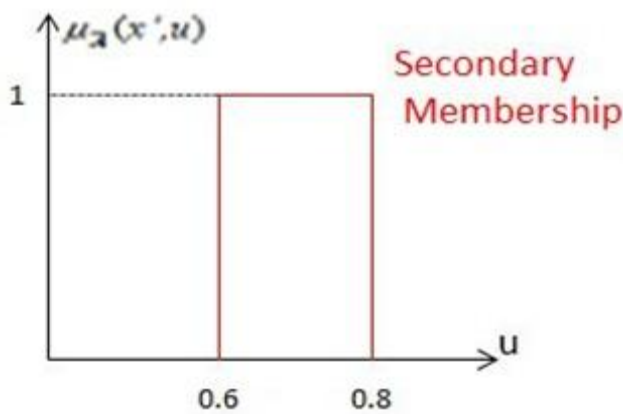


Fig: 2D view of secondary membership for interval type2 fuzzy logic set.

Basically type2 fuzzy logic is subdivided into two type a) General type2 fuzzy logic, b) Interval type2 fuzzy logic. If the value all secondary membership is 1 for all points, we speak of an interval type-2 set and if we are uncertain about a specific quantity, we can use type-2 sets whose degrees of membership which are themselves fuzzy. This is called General type2 fuzzy set [9]. Due to the fuzzy-fuzzy set nature of type 2 fuzzy set, means the primary membership function of type1 fuzzy set is now again represented another membership function (secondary membership function) therefore the general type-2 FLC is computationally very exhaustive and expensive whereas the calculation becomes straightforward in case of IT2-FL which will help us to use a fuzzy controller for real time control problems. Also all details of how to use IT2 FSs in a fuzzy logic system have been worked out whereas in case of General type2 fuzzy logic the real time implementation is still an area of research. Due to this reason we see that most of the applications using type-2 fuzzy logic are based on Interval Type-2 FLCs that use interval type-2 fuzzy sets. The main difficulty in understanding of Type2 fuzzy set is its 3D nature. But while using Interval type 2 fuzzy set the 3<sup>rd</sup>

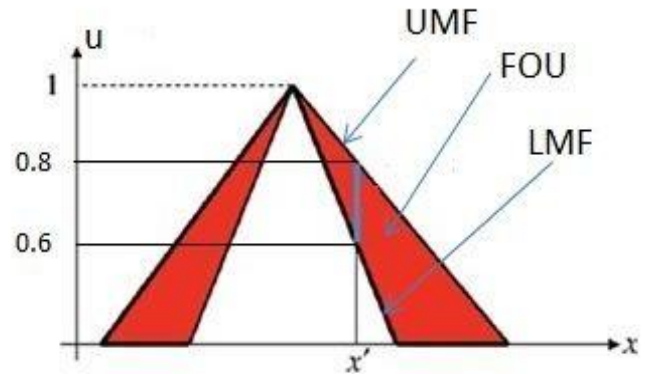


Fig: 2D view of interval type fuzzy logic with FOU area in red shaded area

dimension does not provide any new or additional information. It's the LMF, UMF and FOU area that completely describe the IT2 FLS. Thus we can say that it is the FOU area which helps us to determine the uncertainty in 2D instead of 3D. Also while computing we can say that instead of computing only one type1 fuzzy set now we are computing two type1 fuzzy set one for LMF and other for UMF.

4.1 Type-2 fuzzy logic controller

The IT2-FLC in our case has 3 state variables. Two for input and one for output. The input state variables are  $dir.$  which represents the  $x$  positioning of mobile robot and  $\theta$  which represents the orientation angle of mobile robot. The output state of IT2-FLC is  $\phi$  which represent the steering command to the mobile robot. The  $dir.$  ranges from 0 to 300,  $\theta$  ranges from -150 to 340. Different vehicles have different maximum turning angle ranging from -90 to 90 degree. So in our case we considered the steering angle 38. So  $\phi$  ranges from -38 to 38. The linguistic terms are very common, used in rule base are in fuzzy logic.

Rule Base

Table 1: Fuzzy rules base table

$\theta$ dir.	PB	PS	ZE	NS	NB
PH	NB	NB	NM	NM	PS
PL	NM	NM	NS	PS	PM
ZE	NS	NS	ZE	PS	PS
NL	NS	ZE	PS	PM	PM
NH	PB	PB	PB	PM	PM



Where

- NH, negative high; NB, negative big
- NL, negative low; NM, negative medium
- ZE, zero; NS, negative small
- PL, positive low; ZE, zero
- PH, positive high; PS, positive small
- PM, positive medium
- PB, positive big

**4.2 Type-reducer and defuzzification**

The output processor includes a type-reducer and defuzzifier. The type-reduction method is an extension of type-1 defuzzification obtained by applying the Extension Principle to a specific defuzzification method. The type-reduced set using the centroid can be expressed as [17];

$$Y_{TR} = [y_l, y_r];$$

$$y_l = \frac{\sum_{i=1}^M y_i \mu_{\bar{B}}(y_i)}{\sum_{i=1}^M \mu_{\bar{B}}(y_i)} \quad y_r = \frac{\sum_{i=1}^M y_i \bar{\mu}_{\bar{B}}(y_i)}{\sum_{i=1}^M \bar{\mu}_{\bar{B}}(y_i)}$$

The output of defuzzification step of FLS is obtained by summing the value of  $y_l$  and  $y_r$  obtained from type reduction step and divide it with two, as shown on equation, this will give us the steering value  $\phi$ .

$$y = \frac{y_l + y_r}{2}$$

**5 SIMULATION**

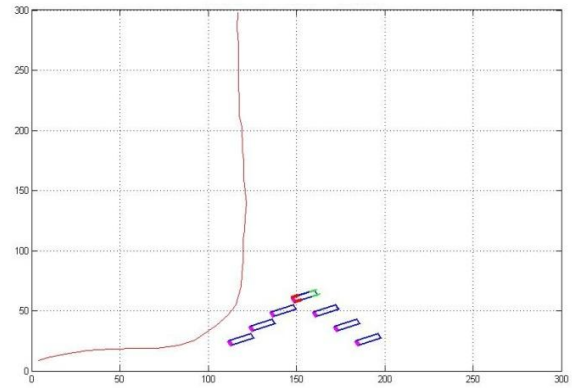
The effectiveness of the fuzzy controller is demonstrated here. This section lays out the simulation environment created in MatLab, and its results. The simulation was designed to match the car as closely as possible. The maximum number of Wheeled Mobile Robot that can be handled by our controller is 7. At first we will enter the number of vehicle we want for our simulation. Once the numbers of vehicles are entered. We will enter the coordinates of first vehicle. The output of simulation is shown below;

enter the No of vehicles : - 7  
 enter position 1 or 2 : - 1  
 enter x location :-10 to 150 : - 150  
 enter y location : - 10 to 60 : - 60

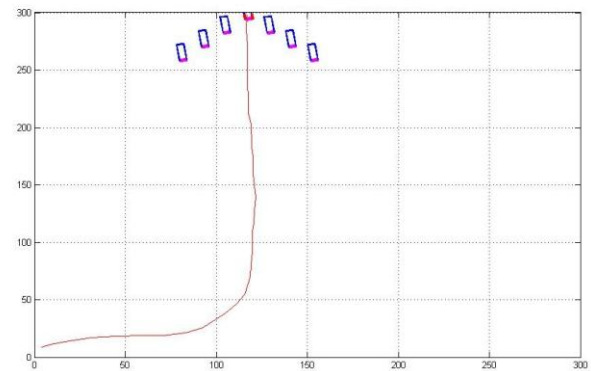
The steering commands  $\phi$  to the leader mobile robot are as follow  
 {19.9146,19.8475,19.7562,19.6457,19.5213,19.5303,18.0106,17.4597,15.9734,14.6912,13.7103,.....-2.9932,-2.8955}

The new positions of midpoint p of robot local body frame are  
 {(153.0927,61.6160),(156.0274,63.5039),(158.7810,65.6473),(161.3325,68.0277),(163.6632,70.6248)(165.755

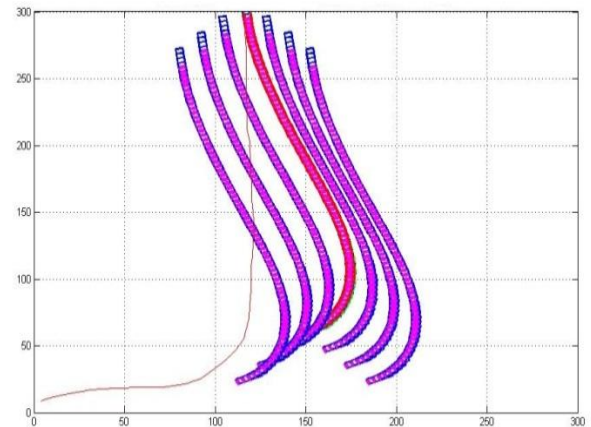
4,73.4176),(167.6037,76.3776),(169.2105,79.4754),(170.5810,2.6850),(171.7291,85.9808).....(117.7147,296.4196), (117.3191, 299.8878)} The initial, final position of group of mobile robots in leader follower formation along with their trail is shown below.



**Fig.6:** Initial position of robots



**Fig.7:** Final position of robots



**Fig.8:** Trail of Robots

**6 CONCLUSION**

The Fuzzy controller shows the motion control of car-like mobile robot very close to realistic motion of car in real world. Mobile robot motion control is a good condition for testing a type-2 fuzzy logic controller because of the uncertainties involved while interacting with the mobile robot moving in its environment in real-time. In this paper, we have shown the

formation strategy of a group of car-like mobile robot in leader-follower structure. We have also given the detailed information about the kinematics of wheeled mobile robot. The efficiency of IT2-FL controller with rule base is also shown for controlling the group of mobile robots moving while they are in formation. This is confirmed by simulation results. The controller is based on simple if and then rules. The future research work will focus on extending the simulation results to more general applications such as employing switching of mobile robots positions, obstacle avoidance, real time continuous path tracking and maintaining formation between leader and follower robots taking into consideration of robot dynamics, model uncertainties.

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