Students' Error Types And Reasoning Ability Achievement Using The Indonesian Realistic Mathematics Education Approach

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Abstract: A teacher needs to compromise the abstract nature of fraction material in order for students to have a better understanding of the concept. The purpose of this research therefore was to find out the type of errors made by students when learning fractions and their achievement of reasoning ability through the Indonesian Realistic Mathematics Education (PMRI). The design of this research was quasi-experimental, involving two schools with one control and one experiment class in each school. One school is private, while the other is public, and classes from each were chosen as experiment and control classes. From these schools, the teaching and learning process using PMRI was only conducted in the experiment classes, while in the control classes, conventional teaching and learning was used. The learning process used concrete objects as the learning media, namely: paper, mortar, and fraction boards. Through the use of fraction board, students can find two simple fractions with similar values. There is a difference in the students' achievement of the mathematical reasoning ability between the implementation of the PMRI and conventional approaches. In general, the students found difficulty in understanding and stating a fraction based on the representation of a unit expressed by image/facts. The students often misinterpret the word "part" not as part of the whole, but as a specific unit based on the number of pieces available without being divided by the whole. The error of doing calculations was the most frequent mistake made by the students and is categorized as the common error category.

Keywords: types of error, reasoning, realistic mathematics education, PMRI, elementary school, fraction, concrete

1. INTRODUCTION

It is hard to imagine how a learning process runs and what the result is for people who do not possess the vision ability while they want to see color, or, for people who have the vision ability but they never see how the color looks. The issue with this kind of learning process is that the abstract nature of the material restricts the internalization of the concept in the students' minds. Teachers, as instructors and facilitators, need to select the appropriate learning approach and to look carefully at students' abilities to minimize errors. That it is necessary for teachers to develop more appropriate learning media, strategies, or models that are more suitable to learning materials or the contexts that their students are dealing with [1]. When the process of finding a mathematical concept is planned and conducted consciously, the activities will give a strong impression for remembering findings [2]. A high quality learning process contributes significantly to the students in finding mathematical concepts. For this reason, teachers are expected to be involved in guiding their students who need assistance and also make sure that they have the initial initial abilities required. Moreover, the fourth-grade students in Indonesia are usually aged 9 to 11 years old. Their cognitive development is at the stage of operational concreteness. It implies that the mathematics learning at this age should utilize manipulatives so that the retention of the concept in the students' mind would be high [3]. Mathematics manipulative materials are objects that can be handled by an individual in a sensory manner. During the handling process, conscious and unconscious mathematical thinking will be fostered [4]. Furthermore, a mathematics manipulative object has the potential to lead to awareness and development of concepts and ideas linked with mathematics, and would most likely have to be purpose designed. Learning to use manipulatives is regarded as a fundamental part of elementary-level mathematics. Many mathematic concepts could be explained better by using manipulatives rather than using traditional learning including fractions [5]. Fractions are first introduced to students when they are in the 4th grade and, therefore, it is important to engage in a proper and joyful learning process that is age appropriate, therefore, students can avoid misconceptions in their understanding. The learning process should begin by understanding the concept of fractions through activities that use a concrete object, as this gives students something to base their understanding on. Once they understand the meaning of a fraction, it is then given a symbol corresponding to what they have seen and understood through the concrete object. Indonesian students, however, still face difficulty in learning fractions. For instance, the restrictive nature of definitions in Indonesian textbooks might make students struggle with understanding fractions as part of a set and also where they fit on a number line [6].

The purpose of a problem that requires reasoning skills is that it requires careful consideration and planning to decide on the solution. Not all given arguments in a problem can be used to find the solution to a problem. According to [7], instructional programs from pre-kindergarten to Grade 12 should enable all students to (1) recognize reasoning and proof as the fundamental aspects of mathematics, (2) make and investigate mathematical conjectures, (3) develop and evaluate mathematical arguments and proofs, and (4) select and use various types of reasoning and methods of proof. Furthermore, defines reasoning as the thinking process which includes the activity of drawing conclusions based on available data and events 8]. The main problem in this research was evaluating the extent of students' achievement and the types of error regarding their mathematical reasoning ability in the PMRI learning method. PMRI is the Indonesian version of realistic mathematics education created by Freudenthal. Freudenthal's theory is that mathematics education is inseparable from human activities. the present form of RME had been mostly determined by Freudenthal's view on mathematics, but also

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that mathematics must be connected to reality, stay close to children's experience, and be relevant to society – to be the human value [9]. PMRI is a learning approach that utilizes contextual and concrete objects, which can be used to understand concepts and problems, and represent fractions based on the context chosen. In connection with this that the learning strategies improve students in developing ideas for solving mathematical problems [10]. Furthermore, that RME also helps teachers to simplify and realize mathematical concepts [1]. Therefore, the teachers need to be more creative and innovative in designing learning methods by using this approach. RME is viewed as a potential approach to increasing students' understanding of mathematics [11]. The PMRI team, inspired by the philosophy of RME, have developed an approach to improve mathematics learning for Indonesian schools. It is known as PMRI, which stands for Pendidikan Matematika Realistik Indonesia or the Indonesian version of Realistic Mathematics Education". The characteristics of PMRI learning in this research also adopted the five features of RME, which are in line with students initial understanding of [12] as presented in Table 1.

Table 1
The characteristics of PMRI learning used in this research

No	Characteristics	Content/Form
1	Using context	Distributing chocolates to some students.
2	Using model	Representing fractions using board and paper.
3	Students' contribution	Using the appropriate students thinking pattern.
4	Interactive	Group discussion utilizing concrete models and the teachers acts as a facilitator to students' need.
5	Intertwining	Rectangle (a shape of the chocolate which consists of 24 smaller pieces with 6 cm length and 4 cm width)

Students' contribution to the learning process is in accordance to their thinking pattern, and communication with friends (within or outside the learning group).

2 METHOD

2.1 Design of the Study

In order to minimize time and maximize resources, qualitative and quantitative data was collected simultaneously [13]. This study was designed using a quasi-experiment with the total of four classes from two schools (private and public institutions). One class from each school was chosen as experiment and another as control. The PMRI teaching and learning process was conducted in the experiment class, while the conventional was carried out in another class. The quantitative data was obtained from students' worksheet, where answers to the proposed problems were analyzed.

2.2 Sample

The experiment was carried out in the 4th grade classes of two elementary schools in Banda Aceh, Indonesia. Table 2 shows the total sample used in this study.

Table 2
The Research Subject

No	School Name	Total Subject		Total
		Eksperiment	Control	
1	SD Negeri 2 Banda Aceh	24 (kelas 4A)	22 (kelas 4D)	46
2	SD IT Nurul Ishlah Banda Aceh	27 (kelas 4B)	23 (kelas 4A)	50
	Total	51	45	96

The class formation in each school was randomly selected by the committee without differing the students' background and ability but based on references from the mathematics teachers. The teachers were considered experienced owing to their participation and training in a number of PMRI courses. The data used in the statistical calculation was obtained from sample classes which followed the testing and non-testing activity, with the total of 96 students (50 from private institutions and 46 from public).

2.3 The Research Instrument

- *Concrete Material*

"Students' skills are obtained by observation, asking questions, reasoning, presentations and creativity" [14]. The teacher needs to utilize a learning approach which is based on problem-solving activities to encourage students' ability to produce contextual work, both individually and in groups. An effort used to facilitate this is to engage them in activities mentioned in the curriculum is by utilizing concrete materials as presented in Figure 1.

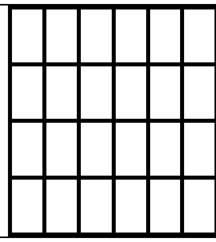
Figure 1. *The concrete materials used in this research*

In Figure 1, the following were utilized (1) rectangle board, (2) paperboard, and (3) mortar to facilitate students thinking pattern. [15] Asked students to engage in a learning using paper strip, while [13] used playdough (plasticine) to represent a fraction.

- *Fraction Board*

Fraction boards can be used by students individually or in groups to represent a variety of simple fractions having factors of 24 as the numerator or denominator. An example is presented in Figure 2.

Chocolate

Chocolate Model (Fraction Square)

Figure 2. *Fraction board similar to a Chocolate*

Furthermore, it can be used to add or subtract two or more fractions. However, before carrying out these mathematical operations, students need to have an understanding and ability to represent values of a fraction using the board. This can easily be carried out using one or several pieces of paperboards model that has been provided to cover the part of the fraction board according to the proposed problem.

- *Paperboard*

The rectangle fraction board is equipped with paperboards with the suitable size to represent various models. Each shape consists of several types of small chocolate-like pieces as shown in Figure 3.

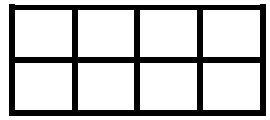
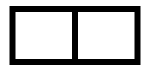
Fractional Finding Discussions Worth Using Paper Cartons

The Sample of Using the Fraction board (represented $\frac{1}{2} = \frac{2}{4}$)

Figure 3. *Students analyzing the concept of similar fractions using paperboards.*

This technique was used to represent the corresponding fraction with two boards given to each group of students in a bid to represent the corresponding piece and carry out simple mathematical calculations. Eight paper sizes represented different values of the fraction: $\frac{1}{2}$ with size 2 x 6 or 3 x 4, $\frac{1}{3}$ with size 2 x 4, $\frac{1}{4}$ with size 1 x 8, or 2 x 4, $\frac{1}{6}$ with size 1 x 4, $\frac{1}{8}$ with size 1 x 3, $\frac{1}{12}$ with size 1 x 2, $\frac{1}{24}$ with size 1 x 1, as well as one size of 4 x 6. This is illustrated in Table 3.

Table 3
The representation of fractions

Value of Fraction	Paper Size as Representation	Paper Form
$\frac{1}{3}$	2 x 4	
$\frac{1}{12}$	1 x 2	

The fraction board model is equipped with paperboards representing chocolate chips model consisting of 24 small pieces as shown in Figure 2. When the students engaged in learning activities using the provided model, it provided an opportunity for them to carry out the process "independently".

- *Mortar*

Mortar is a kitchen utensil made of clay and traditionally used to grind chili, onion, and other spices. It is shown in Figure 4

1 mortar	"Fraction" the broken mortar	Fraction Representation $\frac{1}{4}$ (designed/formed)

Figure 4. *Mortar and Tre Fraction ("$\frac{1}{4}$")*

In the teaching and learning stage, mortar is broken by each group in order to determine concrete fraction from the whole. This is carried out to strengthen students' understanding on fraction as shown in figure 4 and is used to represent a fraction of $\frac{1}{4}$.

2.4 Data Collection

The data was collected through some stages, which include observation, tests, and unstructured interviews technique.

Initial observations were carried out to obtain data on the background and qualifications of the mathematics teachers' as well as the number of fourth-grade classrooms from each school. After the researchers and the school reached an agreement, the process of teaching and learning mathematics with PMRI approach was conducted in the experiment class. The teaching and learning process continues till the teacher exhausts the curriculum. This teaching process was completed with several daily tests to check students' understanding, at the end of each part of the material (sub- material).

a. Instrument Validation

Before the learning instrument was used, it was validated using the Q-Cochran test as well as comments and revisions from the content and language experts with the appropriate background. In addition, a trial (pilot study) was also conducted to the non-participant teachers and students. This process was carried out to ensure the instruments used were appropriate with the students' ability, legibility, language usage (redaction), and to the material concept.

b. Test and Non-Test Instrument

The apparatus used to collect data consist of two forms namely test and non-test instrument. The test instrument tests the initial mathematics ability (KAM) and mathematical reasoning ability (KPM) with the ideal score of 48. The non-test instruments consisted of a questionnaire about students' responses to the learning process, a set of lesson plan, and a teacher observation sheet. The score test was used to measure the students' initial mathematics ability and was also used as the criterion to decide the study group. The test consists of 9 questions which were related to students ability to learn fraction and its operations, such as addition, multiplication, subtraction, and division, while, the test of mathematical reasoning ability (KPM) consists of 5 questions.

2.5 Data Analysis

The data analysed in this research were both quantitative and qualitative. The quantitative data was mathematical reasoning ability test (KPM) score after the teaching and learning process finished. While the qualitative data analysed comprises of various types of errors made by students, the quantitative analysis was carried out to draw conclusion based on the proposed hypothesis. It was used to determine students' achievement, while the qualitative data was used to analyze errors and the learning strategy using the PMRI. Before carrying out the hypothesis test, the normality test of *Shapiro-Wilk* was applied*.* The achievements were analyzed using the t-tet. The data obtained is homogenic on *Mann-Whitney* or heterogenic when normally or not-normally distributed. Indicators for the mathematics reasoning ability are as follows (1) using or interpreting mathematical model such as formula, graph, table, scheme, and drawing conclusions from them; (2) solving problems using appropriate method such as arithmetical, geometrical, or analytical; (3) communicating information effectively using symbols, visual, numerical, or oral representation; and (4) assessing the accuracy level of the conclusion based on the quantity of information. It also comprises of nonroutine problems on fraction.

2.6 Research Limitation

This research was limited to the definition, sorting, simplification, addition, subtraction, mixed operations, and problem-solving related to fractions. "*Elementary school textbooks thoroughly address the part of a whole or set meaning for fractions. Children may have experience working with that idea, but do not get many opportunities to make out sense of the other meanings and interpretations. Education researchers have agreed upon five main interpretations of fractions namely: part of a whole or set, division, ratio, operators, and measurement* [16]. This research focused on defining fraction as a "part of a whole" and through a discrete object, some amount of objects can be measured from it.

3. RESULTS AND DISCUSSION

The Achievement of Mathematics Reasoning Ability

The achievement of the mathematical reasoning ability is presented in Table 4. Description of the final test of the mathematical reasoning ability in all categories

Group		The achievement of mathematical reasoning ability
Experiment	Mean	20.0000
	Deviation standard	10.93801
	N	51
Control	Mean	14.9111
	Deviation standard	8.82615
	N	45

Based on Table 3, the average achievement of mathematical reasoning ability for the experiment group who utilized the PMRI is higher than the average control group who got the conventional technique. Based on the data in Table 3, a statistical test was conducted to generate conclusion, between the PMRI and conventional approaches. The result of the testing is presented in Table 5.

Table 5

The independent sample t-test result

Group	Mean	T	p-value	Result	Conclusion
Experiment	20.00	2.487	0.015	H_0 is rejected	There is difference
Control	14.91				

Based on the results of statistical calculations, a t-value of 2.487 with a p-value of 0.015 were obtained. The statistical test result shows that when the p-value (0.015) < 0.05, H0 is rejected. Therefore, it can be concluded that there is a difference in the achievement between both approaches. The calculation resulted in an average PMRI learning score of 20.00, while the conventional learning is 14.91 which shows that the PMRI approach is better than the conventional learning technique.

The Error Analysis

The answers to each item and sub-item were classified into three as follows: wrong, correct, and incomplete. These were considered to be true as long as the presentation was in line with the context of the problem, through figures/graphics and sentences. Meanwhile, the answer was considered wrong if the item was not answered. Incomplete answers do not provide complete solution to a problem according to its context. In general, students found it difficult to understand and state fractions as a unit of expression. However, they were able to correctly choose a rectangle figure with a certain length and shaded region representing "$\frac{1}{3}$ part of the whole", as shown in Figure 5.

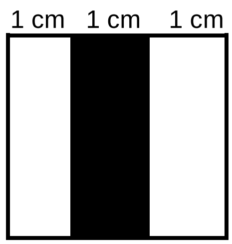


Figure 5. *Representation of " $\frac{1}{3}$ part of the whole" using rectangle model*

However, when the representation was illustrated as shown in figure 6, the students also regarded the shaded region as the $\frac{1}{3}$ part of the whole though it was not true.

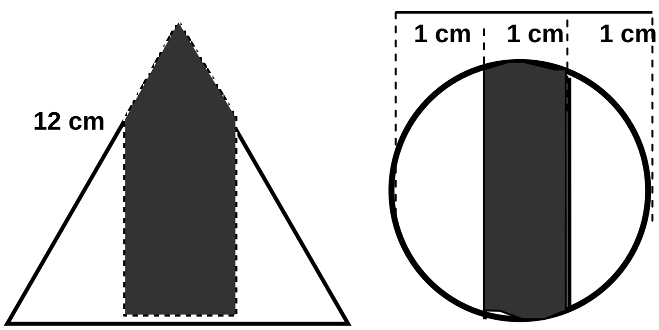


Figure 6. *The wrong representation of $\frac{1}{3}$ using equilateral triangle (left) and circle (right)*

The description shows that the students still found it difficult to represent a fraction. Its understanding is useful in mathematics and everyday life context.Fraction is defined as part of a whole. The use of the word "part" with respect to "how many parts of the whole?" is often interpreted as separation. This is often used in elementary school books to define the value of a fraction. The avoid this misunderstanding, the word n parts should be added as a suffix to "fraction value". Based on the findings of this research, when a concrete object has been divided into n parts, without equal values, the students could misinterpret it as a $\frac{1}{n}$ of the whole. Therefore, the teacher needs to make sure that the sections of the concrete object used are of equal size.

Common Error Category

The most frequent error made by students is miscalculation with over 63 cases of wrong answers in addition/subtraction operation. In adding and subtracting two simple fractions, 21 students could not answer completely and correctly, and four made unnecessary mistakes owing to careless calculation. When two students calculated the contextual problems using chocolate chips, each obtained $\frac{2}{7}$ and $\frac{1}{4}$ part. They were asked the following questions: (1) what information is contained in the word problem, (2) what is asked, and (3) how to solve it. Careless calculation errors were detected when the students converted the fraction to similar form (i.e. $\frac{2}{7}$ to be $\frac{4}{28}$), while it was meant to be correct at ($\frac{1}{4}$ to be $\frac{7}{28}$). The process started by converting the fraction into similar forms to gain the equal denominator, namely $\frac{2}{7} + \frac{1}{4} = \frac{4}{28} + \frac{7}{28}$. The students engaged in a problem which was initially a unit of chocolate represented using the symbol $\frac{28}{28}$. Some parts ($\frac{11}{28}$) of the chocolate were given out, leading to $\frac{28}{28} - \frac{11}{28}$ and resulting in $\frac{17}{28}$. The students have chosen the right strategy to convert $\frac{1}{4}$ to be $\frac{7}{28}$ (They determine the denominator correctly), and the next steps were also correct. However, since the previous step was wrong ($\frac{2}{7}$ to be $\frac{4}{28}$), therefore, the final result was also wrong.

Misconception Error Category

Misconception error means applying an inappropriate knowledge acquired from the previous learning process. The category of misconception in this research consists of three indicators, namely: error of fact, error of concluding, and misunderstanding the problem. Five students were making factual errors in answering the problems. The errors of fact occurred when the students provided different facts with the arguments provided. The instrument to measure students' ability to understand the concept of a simple fraction was arranged in a selected context. The following word problems present how they deal with the concept of "half." Andi and Nurul have their drinking bottle filled with water up to $\frac{1}{2}$ part. It turned out that the water in Andi's drinking bottle was more than Nurul's. Why is that? Please explain. The number of students who gave the wrong answer was 44, while the other 43 students answered correctly. Those that provided wrong answers did not understand the problem, because they felt the bottle was having more water because of its "smaller size." Based on observation, answers were given with different amount of watert. The mistake made was that the students gave different information from what was mentioned in the problem, for example, Andi fills $\frac{1}{4}$ and Nurul $\frac{1}{2}$, even though it was said that both of them filled $\frac{1}{2}$. Error in concluding means that the students give the opposite answer. The third indicator for the category of misconception is "misunderstanding the problem," such as the argument involved in answering the problem which is not relevant. Look at the rectangle ABCD in the picture below (see Figure 7). Check whether the black area (AKD) is $\frac{1}{4}$ part of the rectangle ABCD. Explain your answer using picture or words.

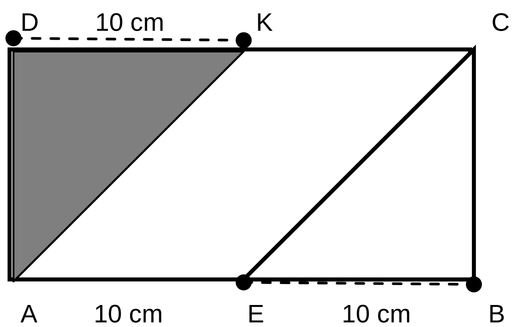


Figure 7. *Rectangle ABCD*

According to some students, "AKD is not $\frac{1}{4}$ because the size is not equal." However, others (code 26 ITEB) failed to understand the problem and responded by saying, "It is not, because it is not divided into four parts and is not equal." Based on the argument, students' understanding of "parts of the whole" is still weak. The student (code 6 NEA) answered: "Yes, the length of the black part is 4 cm." Based on the answer, they misunderstood the problem because the answer given was not relevant. Others were of the opinion that (code 7 NEA) the middle part is not the same as the others without considering it can be made into two congruent right triangles to AKD by making a straight line from point K to E, so the rectangle ABCD consists of 4 congruent triangles.The achievement of the student's final test (code 12 NEA) who were used to deal with nonroutine problems was 48 (excellent). The answer was "Yes because the middle part was divided into two equal parts (right triangle)." This was equipped with an illustration of making an additional line which divides AECK into two congruent triangles with each a $\frac{1}{4}$ symbol.

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