Basic Discrete Systems Analysis Of Water Pipeline Network

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Abstract: A municipal water reticulated pipeline network in Owerri, Nigeria previously analyzed by Simultaneous Loop Flow Correction Method, was re-evaluated using Basic Discrete Systems Analysis Method. This study was carried out to validate the results of the earlier study and decide if the water pipeline network flow characteristics can also be evaluated in a later study for more insight. This study sought, produced a better representation and explanation of the head loss and discharge profiles for the water pipeline network. The water pipeline network consists of twenty-four discrete system (pipe elements) and pipe connections become the nodes which consists of sixteen nodes. These pipeline elements were viewed as discrete systems and flow equations, from first principles, are formulated for each element. Plots were developed to show the nodal head loss, nodal pressure, pipe elements corrected head loss and pipe elements corrected discharge being affected by the pipe diameter and length. The plots revealed that the solutions which were obtained as corrective flow parameters (head loss, pressure and discharge) for the water pipe network described the flow profiles of the pipe network and the flow fluctuations being experienced in the pipe network.

Keywords: Basic Discrete Systems, Pipeline Network, Flow Equations, Elements, Nodes, Head Loss, Pressure

1.0 Introduction

Water is an essential natural endowment that can be used for a wide range of domestic and industrial purposes. For instance, water is used in oil industries, liquid extractions in hydrometallurgical processes, food processing companies and in most domestic activities such as cooking and laundry in our different domiciles. However, the effectiveness/ease of water supply to these aforementioned areas of utilization from its source has been a great challenge for decades of years due to problems associated with poor water pipeline network. These problems are mostly attributed to discharge and head losses that may not be in proportion to the pipe diameters and length. It is against these backdrops that it becomes paramount that a proper design and most especially analysis of municipal water reticulated pipeline networks like that of Owerri in Imo State be carried out particularly if such pipeline network consists of a greater number of pipes as frequently seen in both oil/gas pipelines and water distribution networks. Thus, this study re-examines head losses and discharge for twenty-four pipeline elements as a case previously analyzed with simultaneous loop flow correction method. This study was initiated after these authors discovered short comings in the previous study. The previous study was evaluated using a wrong empirical relation to describe the flow equation and flow parameters. From [2], the empirical relation used to describe the turbulent flow was $h_{f_i} - h_{f_j} = \frac{8fL_kQ_k^2}{\pi^2 gD_k^5}$ instead of $h_{f_i} - h_{f_j} = \frac{8fL_kQ_k^2}{\pi gD_k^5}$. The introduction of π^2 affected the computational procedures and flawed the techniques employed in performing its analysis. Due to the above findings on the previous study, this study was carried out using a Basic Discrete Analysis method. This method views each pipe as a discrete element and the pipe connections as the nodes.

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2.0 Materials and Methods

This study is based on earlier data obtained on the water pipeline network. The flow in the pipeline network was assumed to be turbulent and as such, the empirical relation governing fully developed turbulent pipe flow (Darcy-Weisbach formula) was adopted from [6] for this analysis. The formula is given as

$$P_i - P_j = \frac{8fL_kQ_k^2\rho}{\pi D_k^5} \qquad 2.1$$

where P_i is the pressure at node i (inlet) for element 1, 2, 3,……,16

P_j is the pressure at node j (outlet) for element 1, 2, 3,……,16

f is the Weisbach friction factor = 0.0242

L_k is the length of pipe element 1, 2, 3, ……., 24

Q_k is the volume flow rate of pipe element 1, 2, 3, ……., 24

D_k is the diameter of the pipe element 1, 2, 3, ……., 24

Equation 2.1 was utilized to determine the fluid nodal pressures and the volume flow rate in each pipe. Also, to evaluate the head loss of the water pipeline network, equation 2.1 is modified to

$$h_{f_i} - h_{f_j} = \frac{8fL_kQ_k^2}{\pi gD_k^5} \qquad 2.2$$

where h_{f_i} is the head loss at node i (inlet) for element 1, 2, 3, …….., 16

h_{f_j} is the head loss at node j (outlet) for element 1, 2, 3, ……., 16

g is the gravitational constant = 9.81m/s^2

Table of values (Table 2.1) was imported from [2]. The water pipeline network was relabeled in order to fit into proper assemblage and connectivity. The rearranged data table becomes

Table 2.1. *Table showing values for pipeline network diameter, length and volume flow rate.*

PIPE	1	2	3	4	5	6	7	8
D (m)	0.30	0.15	0.30	0.30	0.30	0.60	0.20	0.15
L (m)	700	300	400	350	300	500	200	200
Q	0.40	0.12	0.695	0.04	0.30	0.20	0.28	0.10

PIPE	9	10	11	12	13	14	15	16
D (m)	0.20	0.15	0.15	0.30	0.30	0.15	0.15	0.15
L (m)	350	200	200	350	800	200	200	200
Q	0.575	0.14	0.14	0.10	0.06	0.18	0.175	0.175

PIPE	17	18	19	20	21	22	23	24
D (m)	0.15	0.15	0.30	0.30	0.30	0.15	0.15	0.30
L (m)	200	300	500	600	350	300	200	600
Q	0.14	0.28	0.04	0.625	0.415	0.21	0.21	0.42

First, equation 2.2 is used to evaluate the initial head loss of the pipe elements using values from table 2.1. The initial head loss values are reported in table 3.1. Next, rearranging equations 2.1 and 2.2 in order to pass them through all twenty-four elements, the characteristic elemental equations are developed. The mass flux rate entering and leaving each element can be derived thus; For determining the nodal pressures, the systems of characteristic elemental equations developed are

For element 1; $q_1 = \frac{1}{R_1}(P_1 - P_2)$ where $R_k = \frac{8fL_k\rho}{\pi D_k^5}$

$q_1 = \frac{1}{R_1}(-P_1 + P_2)$

For element 2; $q_2 = \frac{1}{R_2}(P_2 - P_3)$

$q_2 = \frac{1}{R_2}(-P_2 + P_3)$

For element 3; $q_3 = \frac{1}{R_3}(P_3 - P_4)$

$q_3 = \frac{1}{R_3}(-P_3 + P_4)$

⋮

For element 22; $q_{22} = \frac{1}{R_{22}}(P_{14} - P_{15})$

$q_{22} = \frac{1}{R_{22}}(-P_{14} + P_{15})$

For element 23; $q_{23} = \frac{1}{R_{23}}(P_{11} - P_{15})$

$q_{23} = \frac{1}{R_{23}}(-P_{11} + P_{15})$

For element 24; $q_{24} = \frac{1}{R_{24}}(P_{15} - P_{16})$

$q_{24} = \frac{1}{R_{24}}(-P_{15} + P_{16})$

Next, all twenty-four elemental characteristic equations are assembled into the matrix form of $[Q] = [R][P]$ and its matrix solution is in the form of $[P] = [R]^{-1}[Q]$ 2.3

Equation 2.3 gives the unknown nodal pressure values for $P_1, P_2, P_3, \ldots, P_{16}$. For determining head loss, the systems of characteristic elemental equations developed are

For element 1; $q_1 = \frac{1}{R_1}(h_1 - h_2)$ where $R_k = \frac{8fL_k}{\pi g D_k^5}$

$q_1 = \frac{1}{R_1}(-h_1 + h_2)$

For element 2; $q_2 = \frac{1}{R_2}(h_2 - h_3)$

$q_2 = \frac{1}{R_2}(-h_2 + h_3)$

For element 3; $q_3 = \frac{1}{R_3}(h_3 - h_4)$

$q_3 = \frac{1}{R_3}(-h_3 + h_4)$

⋮

For element 22; $q_{22} = \frac{1}{R_{22}}(h_{14} - h_{15})$

$q_{22} = \frac{1}{R_{22}}(-h_{14} + h_{15})$

For element 23; $q_{23} = \frac{1}{R_{23}}(h_{11} - h_{15})$

$q_{23} = \frac{1}{R_{23}}(-h_{11} + h_{15})$

For element 24; $q_{24} = \frac{1}{R_{24}}(h_{15} - h_{16})$

$q_{24} = \frac{1}{R_{24}}(-h_{15} + h_{16})$

Likewise, all twenty-four elemental characteristic equations are assembled into the matrix form of $[Q] = [R][h]$ and its matrix solution is in the form of $[h] = [R]^{-1}[Q]$……..24 Equation 2.4 produces the unknown nodal head loss values for $h_1, h_2, h_3, \ldots, h_{16}$. To obtain the pressure and head loss for each twenty-four pipe elements, equations 2.1 and 2.2 are used to obtain the corrected discharge Q' for each twenty-four pipe elements respectively. Furthermore, the corrected discharge Q' is used to evaluate elemental pressures and elemental head losses.

3.0 Results

Table 3.1 *Table showing computed values for initial head loss from table 2.1 and using equation 2.2.*

Element	Head Loss, h
1	289.533
2	357.367
3	499.470
4	1.448
5	69.798
6	1.616
7	307.810
8	165.447
9	2271.644
10	324.277
11	324.277
12	9.048
13	7.445
14	536.050
15	506.683
16	506.683
17	324.277
18	1945.662
19	2.068
20	605.887
21	155.828
22	1094.435
23	729.623
24	273.609

The results of the solution to equations 2.3 and 2.4 are listed in tables 3.2 and 3.3. The results of the solution to equations 2.3 and 2.4 are listed in tables 3.2 and 3.3.

Table (3.2.& 3.3) *Table showing computed nodal pressure values for $P_1, P_2, P_3, \ldots, P_{16}$.and nodal head loss values for $h_1, h_2, h_3, \ldots, h_{16}$.respectively.*

NODES	P	h
1	270372.55	40.14696
2	-701566.6	-58.9422
3	5869747	610.6109
4	137531.7	26.28291
5	- 1215091	-111.274
6	- 1261673	-116.019
7	- 8096085	-812.966
8	- 605028.1	-49.414
9	-1549634	-145.466
10	-3515102	-345.928
11	-8917570	-896.783
12	-6432794	-643.545
13	-738377.3	-62.817
14	-5684255	-567.042
15	-18023653	-1825.26
16	-20990679	-2127.72

Table 3.4 *Table showing computed corrected discharge Q', pipe pressure P and pipe head loss h*

Pipe Element	Corrected discharge Q'	Pipe Pressure,P	Corrected Head Loss,h
1	0.23	0.939	95.73
2	0.16	6.232	635.32
3	0.75	5.706	581.65
4	0.29	0.746	76.09
5	0.44	1.473	150.14
6	0.34	0.046	4.67
7	0.12	0.555	56.54
8	0.21	7.158	729.62
9	0.46	14.262	1453.85
10	0.21	7.158	729.62
11	0.19	5.859	597.27
12	0.19	0.320	32.66
13	0.31	1.950	198.74
14	0.12	2.337	238.24
15	0.18	5.259	536.05
16	0.07	0.795	81.07
17	0.12	2.337	238.24
18	0.24	14.023	1429.47
19	0.25	0.792	80.78
20	0.57	4.944	503.94
21	0.49	2.131	217.24
22	0.23	12.879	1312.83
23	0.24	9.349	952.98
24	0.44	2.946	300.29

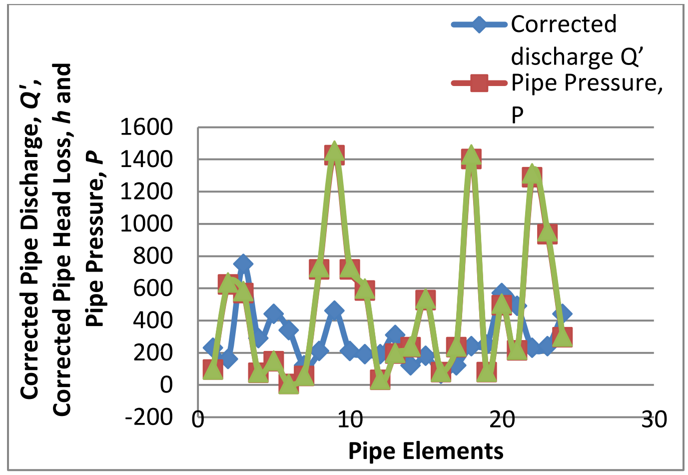

Fig. 3.1 *Plot of Corrected Pipe Elements Discharge, Corrected Pipe Elements Head Loss and Pipe Elements Pressures against Pipe Elements*

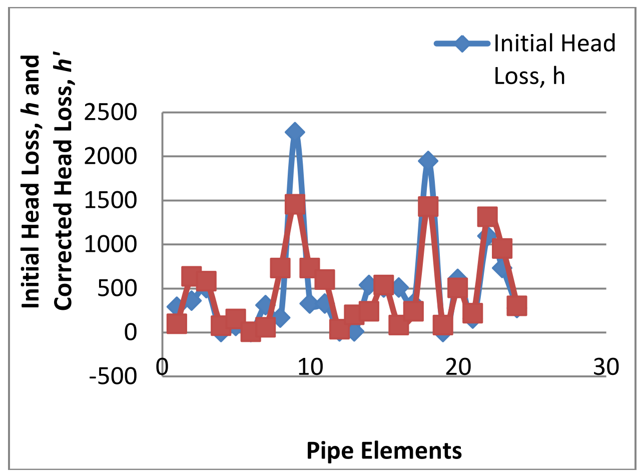

Fig. 3.2 *Plot of Initial Pipe Elements Head Loss and Corrected Pipe Head Loss against Pipe Elements*

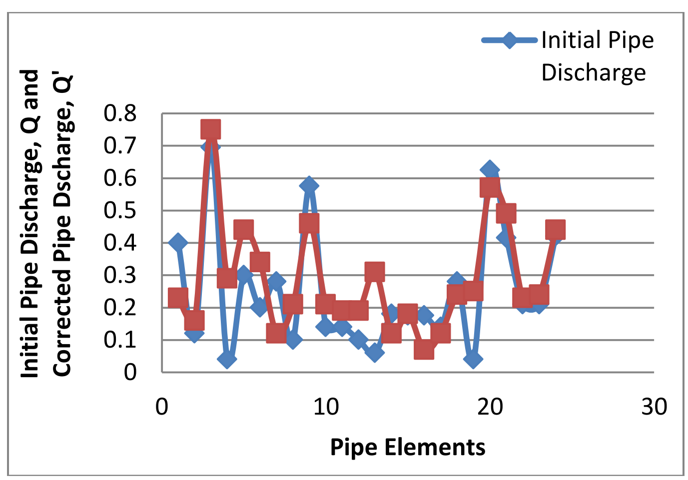

Fig. 3.3 *Plot of Initial Pipe Element Discharge and Corrected Discharge against Pipe Elements*

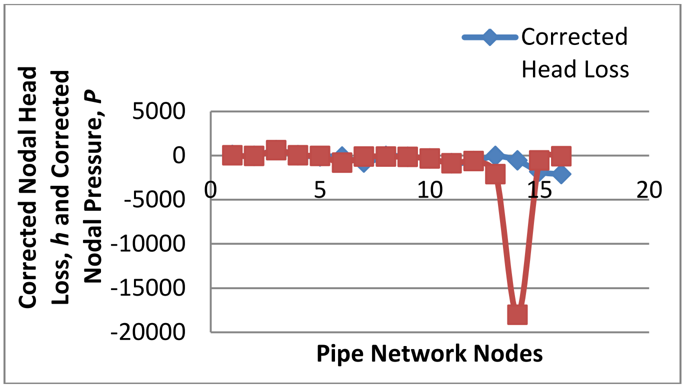

Fig. 3.4 *Plot of Corrected Nodal Head Loss and Corrected Nodal Pressure against Pipe Network Nodes*

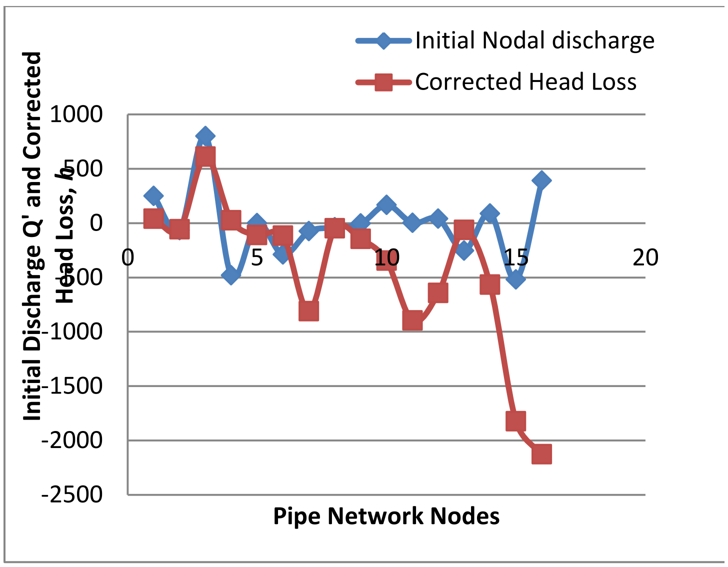

Fig. 3.5 *Plot of Initial Nodal Discharge and Corrected Nodal Head Loss against Pipe Network Nodes*

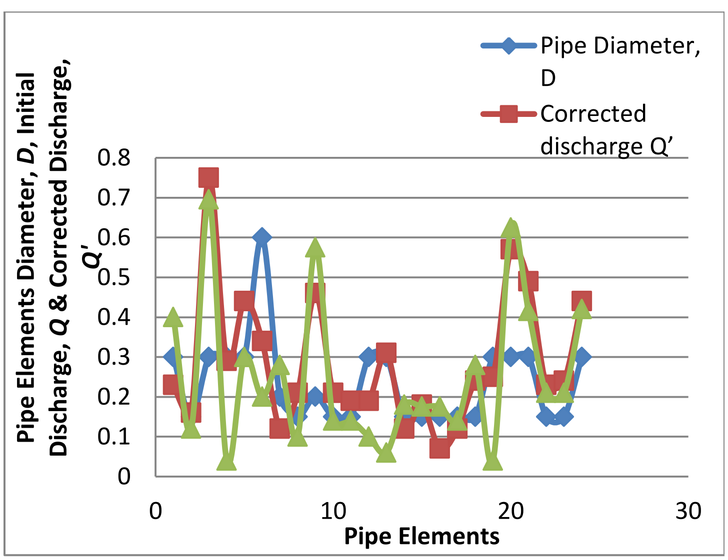

Fig. 3.6 *Plot of Pipe Elements Diameter, Pipe Elements Initial Discharge and Pipe Elements Corrected Discharge against Pipe Elements*

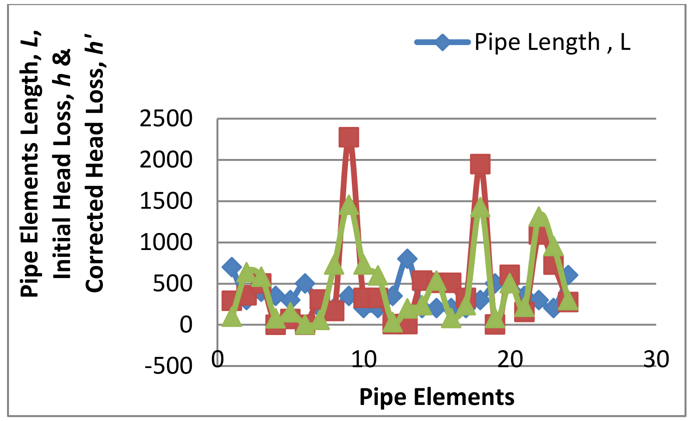

Fig. 3.7 *Plot of Pipe Elements Length, Pipe Elements Initial Head Loss and Pipe Elements Corrected Head Loss against Pipe Elements*

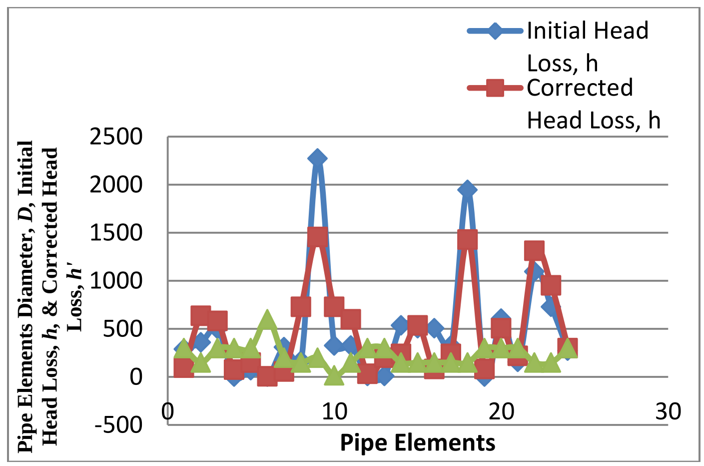

Fig. 3.8 *Plot of Pipe Elements Diameter, Pipe Elements Initial Head Loss and Pipe Elements Corrected Head Loss against Pipe Elements*

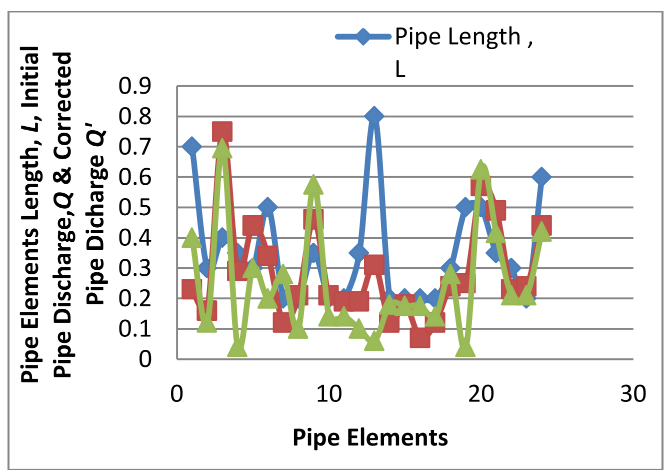

Fig. 3.9 *Plot of Pipe Elements Length, Pipe Elements Initial Discharge and Pipe Elements Corrected Discharge against Pipe Elements*

4.0 Discussions

The solution technique applied to obtain Figs. 3.1 to 3.9 is the basic discrete method. The various plots showed convergence of known pipe parameters to solved parameters. The pipe network showed significant corrections as reported by these plots. Fig. 3.1 reports a comparison of the twenty-four pipe elements against the corrected discharge, pipe pressure and corrected head loss. The graph trends of pipe pressure and head loss in each twenty-four pipe elements are the same. This shows that the flow through the pipe network has the same magnitude in the pipe head loss and pipe pressure. Also, the corrected discharge is within the profiles of both the pipe pressure and corrected head loss except at some elements (1, 3, 4, 5, 6, 13, 21 and 24). Water flow will be reduced because the water pressure is low at some of these pipe elements and not felt in some. Fig. 3.2 reports a comparison between the initial head loss and corrected head loss. The corrected head showed a better profile which is very steep and more coherent than the initial head loss. Element 9 and 18 exhibited huge head losses more in the initial head loss and the corrected head loss reduced these losses greatly. From this plot, the corrected head loss profile will aid more efficient water flow and discharge than the initial head loss. And likewise, from Fig 3.3, elements 9 and 20 exhibited huge discharges more in the initial discharge than the corrected discharge which was also reduced by the corrected discharge as evident in the discharge profile. Fig. 3.4 reports a comparison of the nodal corrected head loss against the nodal pressure against the pipe network nodes. As earlier stated above, this pipe network has twenty-four pipe elements and sixteen nodes. The corrected nodal head loss and corrected nodal pressure also have similar profiles in the same manner as the pipe element head loss and pipe element pressures except for a little misfit. At node 14, the nodal pressure is so high and this makes the head loss very high thereby bring in very turbulent flow regime. As depicted in Fig. 3.5 (which is a snippet of Fig. 3.4) in order to vividly view the nodal discharge and corrected head loss, the initial nodal discharge is within the profile of the nodal pressure discharge but the huge corrected nodal pipe head loss aroused from the high nodal pressure at node 14. This can be attributed to high water flow rate from node 3 through elements 9, 16, 23 and nodes 7, 11 to node 14. Figs. 3.6 to 3.9 depicts comparisons of pipe elements parameters (pipe diameter and pipe length), pipe elements initial discharge, corrected discharge, pipe elements head loss and corrected head loss. The pipe diameter agreed more with the pipe elements corrected discharge than the pipe elements corrected head loss. For the pipe elements corrected discharge, the diameter of pipe elements 1, 6 and 12 were not within the pipe elements corrected discharge profile and the corrected discharge has less pipe elements dimeters outside its profile than the initial discharge. The pipe length agreed more with the pipe elements corrected head loss than the pipe elements corrected discharge. For the pipe elements corrected head loss, the length of pipe elements 1, 4, 5, 6, 12, 13, 19 and 24 were not within the pipe elements corrected head loss profile. Remarkable depicted, the corrected head loss has less pipe elements lengths outside its profile than the initial discharge. The pipe length did not agree with the initial discharge profile and the discharge of water becomes very difficult to people along very long distances on this pipe network.

5.0 Conclusion and Recommendations

5.1 Conclusions

The Basic Discrete Method was successfully applied and solution obtained for a twenty-four elements pipe network with sixteen nodes. The solutions which were obtained as corrective flow parameters (head loss, pressure and discharge) for the pipe network was properly used to evaluate the flow profiles of the pipe network and used to explain the flow fluctuations being experienced in the pipe network. These fluctuations were as a result of the pipe network having very large pipe lengths at very low pipe pressures and head loss or large diameters at low discharge rate. For efficient water distribution network to be obtained from this pipe network, water pumps should be installed at certain points to increase flow pressure to some pipe elements and some pipe diameters be increased or decreased at some pipe elements.

5.1 Recommendations

Hence, the Basic Discrete Method is a useful aid in designing pipe network for water distribution for efficient delivery and maximum economy. The authors hope the findings and solution techniques can be applied to other areas of pipeline network. This can also be replicated anywhere in the world. Also, the flow characteristics of a pipeline network can be evaluated to understand the flow regimes of the water pipeline network.

6.0 References

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