Application Of Markov Chain To The Assessment Of Students' Admission And Academic Performance In Ekiti State University

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Abstract: This paper studies the pattern of students' enrolment and their academic performance in the Department of Mathematical Sciences (Mathematics Option) Ekiti State University, Ado – Ekiti, Nigeria. In this paper, A transition matrix was developed for ten consecutive academic sessions. The probabilities of absorption (Graduating and Withdrawal) were obtained. Also fundamental matrix was obtained to determine the expected length of students' stay before graduating. Prediction was made on the enrolment and academic performance of students.

Keywords: Markov chain, Enrolment, Prediction, fundamental Matrix and Probability of Absorption.

Introduction

In probability theory, a **stochastic process**, or sometimes **random process**, is the counterpart to a deterministic process or deterministic system. Instead of dealing with only one possible reality of how the process might evolve under time (as is the case, for solutions of an ordinary differential equation in a stochastic or random process there is some indeterminacy in its future evolution described by probability distributions. Markov process can also be defined as a stochastic process with the property that given the value of x_t, the probability of x_{t+s}, where s>0 is independent of x_u, u<t. That is, the conditional distribution of the future x_{s+t} given the present x_t and the past x_u, U<t, is independent of the past. If the number of states is finite or countably infinite the Markov Process is a Markov Chain. Application of Markov Chain has advanced tremendously in various branches of natural sciences, Engineering and medical Sciences. It is also useful in the development of a model to study the movement of students' in through and out of higher institution. its application provides a means for projecting the number of students graduation and dropout by age, gender and broad field of study. The model also provides estimates of the average time a student stays in the system and the probability of a student completing a course and the average time a student takes to complete a course.Gani (1963), used the application of in out-put models to project enrolment for, and award of, bachel or degrees in Australian Universities. Pollard (1970) also used a version of this model to look at higher education in Australian. Stone (1971: 1972a) wrote extensively on their use in economics, health and education planning. He also applied the model to accounting of pollution (1972b) the in put – out put model was used to study the supply of secondary school teachers in Victoria by Burke(1976) Johnstone and Philip in their paper, Mathematical models can assist educators in the preparation of their educational plans and their potentials in this regard are being increasingly realized. As a result, models have found application at all levels at which planning is conducted. Markov Chain is capable of predicting enrolments for an education system. The model is applied to the new South Wales State Government education system between 1947 and 1961 and the projected enrolments compared to the actual enrolments in those years. Geary (1978), analyses the demand for and the use of educational indicators with reference to Swaziland. The data requirements are considered as well as the policy implications of establishing a fixed series, and an educational input – output table is constructed in an attempt to derive an all- embracing set of indicators. The Markov model presented here was initially created in order to help forecast what might happen to the Swaziland education system rather than to describe it.

DATA SOURCE

The data used for this research work was Students enrolment into the department of mathematical Sciences (Mathematics Option) Ekiti State University, Ado – Ekiti. For a period of ten academic sessions (1997/1998 – 2006/2007) collected from the records Department of the University.

SUBJECT

Students enrolment and their performances were observed, students performances include Enrolment, promoted or Graduated, Repeated and Withdrawn. Estimation of transition probabilities, the entries of the fundamental matrix M, estimation of probability of absorption(Withdrawal and Graduation) and computation of n- step transition probabilities are necessary for any enrolment of students so that the research findings will at best be certain, unbiased and correct. Data was analyzed using Matlab Software. In estimating the transition probabilities, entries of the fundamental Matrix M, estimation of probability of absorption computation of n- step transition probabilities.

RESULTS

A total of 300 students enrolled in fisrt year, 348 in second year, 281 in third year, and 251 in forth year for the ten academic sessions with various categories of graduated, repeated and withdrawn The table below shows the summary of the students' enrolment and their performances over years which is Appendix 1

	1st Year	2nd Year	3rd Year	4th Year
P	114	140	125	0
R	175	193	140	96
W	11	15	16	07
G	0	0	0	0
Ni	300	348	281	251

WHERE
P ---- Promoted
R ---- Repeated
W ---- Withdrawn
G ----- Graduating

This is the distribution of information on the summary sheet in markovian matrix

4.2 ESTIMATION OF TRANSITION PROBABILITY MATRIX

From the table above, we can develop our one- step transition probability matrix as

$$n_{ij} = \begin{pmatrix} & W & G & 1L & 2L & 3L & 4L \\ W & 49 & 0 & 0 & 0 & 0 & 0 \\ G & 0 & 148 & 0 & 0 & 0 & 0 \\ 1L & 11 & 0 & 175 & 114 & 0 & 0 \\ 2L & 15 & 0 & 0 & 193 & 140 & 0 \\ 3L & 16 & 0 & 0 & 0 & 125 & 140 \\ 4L & 7 & 148 & 0 & 0 & 0 & 96 \end{pmatrix}$$

Now, to deduce the transition probability matrix (one step matrix), we divide each element of the row by its row total

$$p_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.0367 & 0 & 0.5833 & 0.3800 & 0 & 0 \\ 0.0431 & 0 & 0 & 0.5546 & 0.4023 & 0 \\ 0.0569 & 0 & 0 & 0 & 0.4448 & 0.4982 \\ 0.5896 & 0 & 0 & 0 & 0.3825 & 0.0279 \end{pmatrix}$$

The one step transition probability matrix p_{ij} above can be decomposed into $Q_i R$ and Q_j which are defined as

$$Q = \begin{pmatrix} 0.5833 & 0.380 & 0 & 0 \\ 0 & 0.5546 & 0.4023 & 0 \\ 0 & 0 & 0.4448 & 0.4982 \\ 0 & 0 & 0 & 0.3825 \end{pmatrix}$$

$$R = \begin{pmatrix} 0.0367 & 0 \\ 0.043 & 0 \\ 0.056 & 0 \\ 0.027 & 0.5896 \end{pmatrix} \qquad O = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

CALCULATING THE ENTRIES OF THE FUNDAMENTAL MATRIX M

Before writing the formula for calculating the entries of the fundamental matrix $M = (I - Q)^{-1}$. Let us consider the associated theorem and its proof.

Where

M is the fundamental matrix

I is the identity

Q is a Matrix

THEOREM

Given,

$$P = \begin{pmatrix} I & 0 \\ R & Q \end{pmatrix}$$

$$Then\ \ M = (m_{ij})\ is\ given\ by$$

$$m_{ij} = \begin{cases} \dfrac{1}{1 - p_{ii}}, & if\ \ i = j \\ \dfrac{\underset{r=1}{\overset{i-1}{\pi}} p_{rr+1}}{\underset{r=1}{\overset{i}{\pi}} I - p_{rr} + I}, & if\ \ i \neq j \\ 0,\ elsewhere & \end{cases}$$

Proof

We proof his theorem by induction method. Let $n \times n$ matrix $Q \; and \; M$ be denoted by $Q_n \; and \; M_n$ respectively. Let ,

$$Q_{ii} = I - p_{11}$$
$$b_{11+1} = -p_{11} + I$$
$$for \quad n = 2$$
$$Q_2 = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$
$$= \begin{pmatrix} p_{11} & p_{12} \\ 0 & p_{22} \end{pmatrix}$$
$$(I - Q_2) = \begin{pmatrix} 1 - p_{11} & 1 - p_{12} \\ 0 & 1 - p_{22} \end{pmatrix}$$
$$= \begin{pmatrix} a_{11} & b_{12} \\ 0 & a_{22} \end{pmatrix}$$
$$|I - Q_2| = a_{11}a_{22}$$
$$\therefore (I - Q_2)^{-1} = M_2$$
$$= \frac{1}{a_{11}a_{22}} \begin{pmatrix} a_{22} & -b_{12} \\ 0 & a_{11} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{a_{11}} & \frac{-b_{12}}{a_{11}a_{22}} \\ 0 & \frac{I}{a_{22}} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{I}{I - p_{11}} & \frac{p_{12}}{(I - p_{11})(I - p_{12})} \\ 0 & \frac{I}{I - p_{22}} \end{pmatrix}$$

Since the statement is true for $n = 2$, it will be also true for $n = 3$

$$Q_3 = \begin{pmatrix} P_{11} & p_{12} & 0 \\ 0 & p_{22} & p_{23} \\ 0 & 0 & p_{33} \end{pmatrix}$$

$$(I - Q_3) = \begin{pmatrix} a_{11} & b_{12} & 0 \\ 0 & a_{22} & b_{23} \\ 0 & 0 & a_{33} \end{pmatrix}$$

$$|I-Q_3| = a_{11}a_{22}a_{33}$$

$$(I-Q_3) = M_3 = \frac{I}{a_{11}a_{22}a_{33}}\begin{pmatrix} a_{22}a_{33} & -b_{12}a_{33} & b_{12}b_{13} \\ 0 & a_{11}a_{32} & -a_{11}b_{23} \\ 0 & 0 & a_{11}a_{33} \end{pmatrix}$$

$$(I-Q_3) = \begin{pmatrix} \frac{1}{a_{11}} & \frac{-b_{12}}{a_{11}a_{22}} & \frac{b_{12}b_{23}}{a_{11}a_{22}a_{33}} \\ 0 & \frac{1}{a_{22}} & \frac{-b_{23}}{a_{22}a_{33}} \\ 0 & 0 & \frac{1}{a_{33}} \end{pmatrix}$$

Then, the 3 X 3 fundamental entries can be calculated as below

$$(M) = \begin{pmatrix} \frac{1}{1-P_{11}} & \frac{p_{12}}{(1-p_{11})(1-p_{22})} & \frac{p_{12}p_{23}}{(1-p_{11})(1-p_{22})(1-p_{33})} \\ 0 & \frac{1}{1-p_{22}} & \frac{p_{23}}{(I-p_{22})(I-p_{33})} \\ 0 & 0 & \frac{1}{1-p_{33}} \end{pmatrix}$$

The statement is true for $n=3$, $assume\ for\ n=k$ it is also true for $n=n+k$

$$Q_{k+1} = \begin{pmatrix} p_{11} & p_{12} & 0 & \ldots & 0 & 0 \\ 0 & p_{22} & p_{23} & \ldots & 0 & 0 \\ . \\ . \\ . \\ 0 & 0 & 0 & & p_{kk} & p_{kk+1} \\ 0 & 0 & 0 & & 0 & p_{kk+1} \end{pmatrix}$$

$$(I-Q_{k+1}) = \begin{pmatrix} a_{11} & b_{12} & 0 & \ldots & 0 & 0 \\ 0 & a_{22} & a_{23} & \ldots & 0 & 0 \\ . \\ . \\ . \\ 0 & 0 & 0 & & a_{kk} & b_{kk+1} \\ 0 & 0 & 0 & & 0 & a_{k+1k+1} \end{pmatrix}$$

$$|I-Q_{k+1}| = a_{11}a_{22}a_{33}\ldots a_{kk}a_{k+1}k+1$$
$$= \underset{r=1}{\overset{k+1}{\pi}} a_{rr}$$

The transpose of the cofactor $T\ of\ (I-Q_{k-1})$ is given by $T=T_{ij}$

$$T_{ij}=\begin{cases}\underset{r=1}{\overset{r-1}{\pi}} a_{rr} \underset{r=i+i}{\overset{k+1}{\pi}} a_{rr},\ if\ \ i=j \\ \underset{r=1}{\overset{j-1}{\pi}}(-b_{rr+1} \underset{r=1}{\overset{t-1}{\pi}} a_{rr} \underset{r=t+1}{\overset{k+1}{\pi}} a_{rr}),\ if\ \ i\prec j \\ 0,\ if\ \ i\succ j \end{cases}$$

$$but\quad M_{k+1}=\frac{T_{ij}}{|I-Q_{k+1}|}$$

$$M_{ij}=\begin{cases}\frac{1}{Q_{ij}},\ i=j \\ \frac{\underset{r=1}{\overset{j-I}{\pi}}(-b_{rr+I})}{\underset{r=1}{\overset{j}{\pi}} Q_{rr}},\ \ if\ \ i\prec j \\ 0,\ if\ \ i\succ j \end{cases}$$

Hence the result

$$M_{ij}=\begin{cases}\frac{1}{1-p_{11}},\ i=j \\ \frac{\underset{r=1}{\overset{j-I}{\pi}} p_{rr+I})}{\underset{r=1}{\overset{j}{\pi}}(1-p_{rr})},\ \ if\ \ i\prec j \\ 0,\ if\ \ i\succ j \end{cases}$$

For easy computation, the entries of the fundamental matrix can be written in the form,

$$M_{ij}=\begin{cases}\frac{1}{1-p_{11}},\ i=j \\ a_1 m_{ij-1},\ \ if\ \ i\prec j \\ 0,\ if\ \ i\succ j \end{cases}$$

$$where\quad a_1=\frac{p_{1-Ij}}{1-p_{11}}$$

To demonstrate this result, let us consider the entries of the fundamental matrix m .

$$m_{11} = \frac{1}{1 - p_{11}}$$

$$m_{12} = \frac{p_{12}}{(1 - p_{11})(1 - p_{22})} = a_2 m_{11}$$

$$m_{13} = \frac{p_{12} p_{23}}{(1 - p_{11})(1 - p_{22})(1 - p_{33})} = a_3 m_{12}$$

$$m_{1k} = \frac{p_{12} p_{23} p_{34} \cdots p_{(k-1)k}}{(1 - p_{11})(1 - p_{22})(1 - p_{33}) \ldots (1 - p_{kk})} = a_k m_{lk-1}$$

Thus, the same procedure is applicable to 2, 3,…,k row of the fundamental matrix M. with the use of Mat lab Software we were able to get Fundamental Matrix M

FUNDAMENTAL MATRIX

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2.3998 & 2.0474 & 1.4836 & 1.1970 \\ 0 & 2.2452 & 1.6269 & 1.3126 \\ 0 & 0 & 1.8012 & 1.4532 \\ 0 & 0 & 0 & 1.6194 \end{bmatrix}$$

PROBABILITY OF ABSORPTION (WITHDRAWAL AND GRADUATION)

The probability that the process will enter the j[th] absorbing state if it starts in the i[th] transient state is called the probability of absorption. It is given as $B = MR$ *where* M is the fundamental matrix. R *is* $(N - K) * K$ Matrix showing the probability of transition from a transient state to an absorbing state. Then b_{ij} is the $(i, j)th$ entry of matrix B.

i	100L	200L	300L	400L
b_{nw}	0.2940	0.2260	0.1430	0.0452
b_{ni}	0.7060	0.7740	0.857	0.9548

FORECASTING THE FUTURE PERFORMANCE OF THE STUDENTS

for $n = 1$

	100L	200L	300L	400L
100L	0.5833	0.3800	0	0
200L	0	0.5546	0.4023	0
300L	0	0	0.4448	0.4982
400L	0	0	0	0.3825

$for \quad n=2$

	100L	200L	300L	400L
100L	0.3402	0.4324	0.1529	0
200L	0	0.3076	0.4021	0.2004
300L	0	0	0.1978	0.4122
400L	0	0	0	0.1463

$for \quad n=3$

	100L	200L	300L	400L
100L	0.1985	0.3691	0.2420	0.0762
200L	0	0.1706	0.3026	0.2770
300L	0	0	0.0880	0.2562
400L	0	0	0	0.0560

$for \quad n=4$

	100L	200L	300L	400L
100L	0.1158	0.2801	0.2561	0.1497
200L	0	0.0946	0.2032	0.2567
300L	0	0	0.0391	0.1418
400L	0	0	0	0.0214

PREDITING THE FUTURE ENROLMENT OF STUDENTS

Given an initial vector which contains the current enrolment of students in a four years academic programme, the future performance can be predicted by (3.7.1)

$$P^{(n)} = p^{(0)}\, p^{*(n)} \qquad (3.7.1)$$

If we take the new students into consideration, then the total enrolment of students at the beginning of the nth academic year will be given as:

$$P^{(n)} = p^{(0)}\, p^{*(n)} + r(n) \qquad (3.7.2)$$

Where $p^{(n)}$ is the state of the cohort of students at the beginning of the nth year.

$P^{(0)}$ is the initial vector

$p^{*(n)}$ is the matrix of transition probabilities after removing the absorbing states at nth year.

PREDICTING THE FUTURE ENROLMENT AND PERFORMANCE FOR THE SESSION 2007/2008

Academic session	1^{st} Year	2^{nd} Year	3^{rd} Year	4^{th} Year
2007/08	26	41	37	39
2008/09	15	33	33	33
2009/10	9	24	28	29
2010/11	5	17	22	25

DISCUSSION OF THE RESULTS

It is discovered from the transition probability matrix that the rate of withdrawal decreases as the students progress to highest levels. The movement of students in a particular level depends on the previous level occupied by individual. Again students performances improve over time as they move from one level to another. This may be as a result of the fact that they understand the system better as they pass from one level to another. It is often very high in 1st year because most of the students are not stable. In essence, change of environment, inability to understand their new environment and tenets of academic work often contribute to their instability. Finally, present students enrolment help to give the insight of the minimum number of students that will enroll in each level in few years to come. These results can be extended to its cohort universities for prediction all things being equal. A good academic programme should have the probabilities of withdrawal being non- decreasing function if iteming to zero while probabilities of graduation should be a non – decreasing function if I approaches unity. This means that prospects should increase as one approaches graduation.

CONCLUSION

Markov Chain Model or input – Out model is very good in education planning. The models show movement of students in through out of tertiary institution. It is useful in projecting the number of students that graduates. It gives the average students that graduate. It gives the average time a student will stay and complete a course of study and also gives the average students that graduate. Application of Markov Chain Model can be implored by policy maker's government agencies to check with respect to a particular educational policy of the institution. Having estimated the future minimum enrolment the school management will be able to adjust when necessary.

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