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Approach To Realistic Mathematics Education In Teaching Mathematics: A Case Of Cosine Theorem – Geometry 10

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Abstract: Increasing the practicality in mathematics education has been a trend of most countries in the world. With this trend, students find that mathematics is not an abstract subject far removed from reality. One of the pioneers in teaching mathematics to be associated with real-life is Hans Freudenthal - Dutch mathematics educator. He developed the theory of Realistic Mathematics Education (RME) with the idea that teaching mathematics must start a realistic situation; today this theory has been widely applied not only in the Netherlands but also in many parts of the world. In order to approach RME into mathematics education in Vietnam, we developed the RME-based teaching model and conducted research with the teaching of cosine theorem - Geometry 10 to verify its feasibility. The results showed that if the teacher used RME – based teaching, students could learn cosine theorem by abstracting and generalizing from a real situation; thereby, they found that mathematics is very close to life, creating a motivation for learning mathematics for them. From this, we can assert that teaching with the RME approach meets the goal of mathematics education in Vietnam in the present and upcoming period in the 21st century, and it needs to be studied and widely applied in mathematics teaching in Vietnam.

Index Terms: RME, Realistic Mathematics Education, Cosine theorem, mathematics education in Vietnam, Teaching geometry, Mathematics in Context, RME-based teaching

1. INTRODUCTION

Realistic Mathematics Education (RME) is a theory of mathematics education, developed in the Netherlands by H. Freudenthal since 1971. The characteristics of RME are real and rich situations, given the critical position in the learning process. These situations serve as a starting source for the development of mathematical knowledge and as a context in which students can apply their mathematical knowledge, then gradually become more formal and general and have less specific contexts. RME has the following two main points ([1], [2]):

1. Mathematics is considered as a human activity;
2. Meaningful mathematics is built from a rich context.

At RME, the situations given to students can come from the real world, or from the fantasy world from fairy tales, or the official math world, as long as the problems are real in mind of students. The teaching process is based on the theory of RME shown in Figure 1 [9].

1.1 Mathematization: horizontal mathematization and vertical mathematization

Mathematics should not be studied as a closed system, but rather an activity of mathematization. These two claims of Freudenthal were clearly formulated by Treffers (1987) [13] in the context of education: horizontal mathematization and vertical mathematization. In horizontal mathematization, students find mathematical tools that can help organize and solve a problem in a real situation, for example, activities such as identifying or describing mathematics Students map, form and visualize the problem in different ways; they explore relationships, recognize similarities in different problems, translate a real-world problem to a known mathematics problem. On the other hand, vertical mathematization is the process of reorganization in the mathematical system itself. The following operations are examples of vertical mathematization: representing a relationship in a formula, stating rules. Gravemeijer (1994) stated that both horizontal and vertical mathematization take place to develop the basic concepts of mathematics or the official mathematical language (see Figure 2).

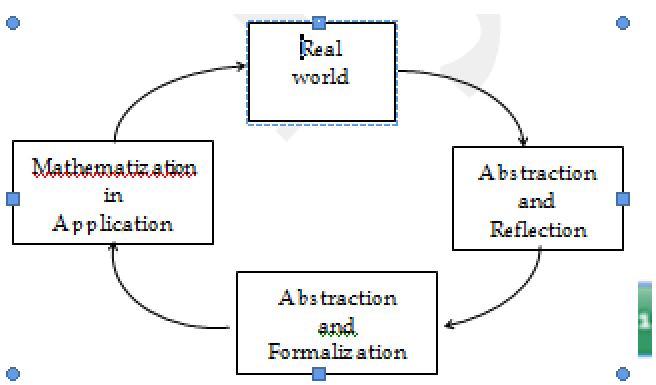

Figure 1. *Teaching cycle based on RME (De Lange, 1996) [9]*

1.2 Guided reinvent

1.3 ion (guided discovery)

The beginning of the reinventing of mathematical knowledge is when the teacher provides the student with a contextual problem related to the subject the teacher wants them to investigate. Instruction should not start with the formal mathematical system and end with an application or problem from the relevant context as an adjunct to be studied after

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appropriate mathematical knowledge has been learned ([4], [5], [10], and [11]). On the contrary, from practical situations in which concepts appear, they can be considered as the basis for forming concepts. Using models or using vertical mathematization tools in problem-solving, students develop and use models as a bridge between abstraction and reality. Initially, it may be a model of a situation familiar to the student. Encourage students to compare their solutions in class discussions. Discussions refer to explaining situations outlined in practical issues, helping students produce more specific things on their own and developing their own informal problem-solving strategies ([4],[6]).

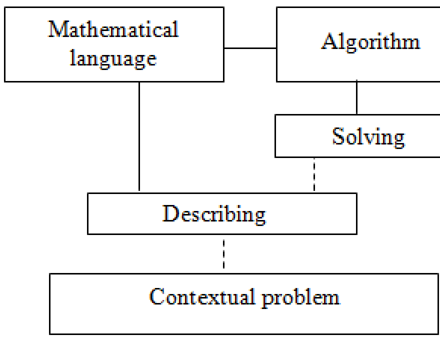

Figure 2: *Process of mathematization* (Gravemeijer, 1994)
Horizontal mathematization (– – –); Vertical mathematization (———)

The role of students in RME is that they work individually or in a team, which means that students are free to make discoveries at their own level, to build knowledge from their own experience. In addition to student--student interaction, student-teacher interaction is also an essential part of RME, and students need teachers to confirm their answers are right or wrong or to be instructed in the mathematical process to lead to a suitable solution for a practical situation, and with the support of a teacher, students make a generalization to come up with mathematical knowledge that they need to learn

1.3 Didactical phenomenology

Freudenthal said: "Students need to learn to imitate the way humanity has gone through; we should not ask students to start learning pre-existing knowledge" [3]. In 1983, Freudenthal introduced didactical phenomenology: phenomenology is a description of phenomena, facts, and circumstances that produce mathematical concepts, structures or ideas. The didactic term refers to the teaching method and teachers' how to teaching. ([3], [7]). Teaching phenomenon is a reminder for teachers to find and exploit phenomena in practice as a means or bridge for knowledge learning, a way to show teachers where learners can step into the process of learning of humanity.

1.4 Research Problem

At the present time, in Vietnam, there have been two textbooks for teaching geometry 10 - high school. The introduction of Cosine theorem in the triangle was presented by the authors in two different approaches. The "Geometry 10 – Basic" textbook, this theorem is expressed according to the deductive approach (firstly, expressing theoretical content; secondly, presenting the proof; finally, giving exercises for applying). In the same case, the "Geometry 10 – Advanced" textbook use the approach from "special case" to "generalization": at the beginning, the book examines the Pythagoras theorem and then introduces the cosine theorem as an extension of the Pythagoras theorem. It is easy to see that the two ways of teaching as above make students not see the fact that mathematics and reality are connected to each other. In the new educational trend in Vietnam, one of the main ideas of mathematics education in schools is to increase practicality in teaching mathematics. The question is whether teaching according to the new trend in teaching mathematics is feasible or not. In this study, we approach RME in teaching mathematical content, which is also a tryout of teaching mathematics in the direction of enhancing the reality of teaching mathematics content in schools.

Research Focus

Based on the RME theory, we developed a teaching model (RME-based Teaching) that starts from a real situation to gradually move toward mathematical knowledge. This model was applied in teaching cosine theorem in Geometry 10, according to the mathematics program of high schools in Vietnam to verify its feasibility.

Research Aim

Applying the RME-based teaching to teaching a mathematical theorem; thereby, evaluating students' ability to move from a practical situation to a mathematical theorem. Also, examining students' ability to perform, abstract and generalize while building specific and general models of the given situation.
Research Questions
This research was conducted with two research questions as follows:
1. If approaching RME theory into teaching mathematics, what will the RME-based teaching covers?
2. Verify whether the way of using the RME- based teaching model that has been developed into teaching cosine theorem - Geometry 10 is feasible or not?

2 RESEARCH METHODOLOGY

2.1 General Background

RME emphasizes that learning math means that students experience a different understanding from solutions related to the context at an informal level, through the creation of different levels, to gain insight into how concepts and strategies are related. Models are essential to bridge the gap between mathematics that is more relevant to the context and more formal mathematics. To perform this bridging function, models have to vary from one model of a specific situation (model of) to a standard model for all other types of situations, but equivalent (model for); in particular, the process includes the following levels.
- Situation level in which knowledge and strategies specific to the knowledge domain is used in the context of the situation.

- The specific model level in which the models and strategies address the situation described in the problem

- The general model level where the focus is on the choice of solving strategies based on mathematical knowledge rather than on specific contexts.

- The formal level (academic) is the degree to which a person who performs mathematical activities with mathematical knowledge stops ordinary ways and symbols.

How to design a lesson with RME

Based on the points of RME as above, we propose how to design a lesson in the RME spirit as follows (see Figure 3).

1. Search for mathematical phenomena in practice (called "mathematical phenomena") related to the content of knowledge to be taught.
2. Select a mathematical phenomenon to design a realistic situation for the lesson.
3. Design teaching process to help students build a specific model (model of).
4. Design teaching process to help students build a general model (model for).
5. Lead students to the mathematical knowledge they need to learn.
6. Select exercises for students to apply (including pure math exercises or practical problems).

Model of RME - based Learning

From the fundamental points of RME, we develop the RME-based teaching, which consists of 7 steps as below.

Step 1: Teacher presents a realistic situation.

Step 2: Students learn the situation clearly (vocabulary, context, terminology, etc.).

Step 3: Students describe the situation in the form of a specific model (model of).

Step 4: Students find answers to problems raised by the situation.

Step 5: Students make generalizations to convert a specific model (model of) to a general model (model for).

Step 6. Students learn mathematical knowledge on the basis of a general model.

Step 7: Teachers let students practice applying the knowledge they have just learned.

Table 1 makes clear about the role of the steps of RME – based teaching

Table 1: *The role of the steps of the RME – based teaching model*

Realistic situation	Horizontal mathematization	Vertical mathematization	Application
Step1, step2	Step 3, Step 4	Step 5, Step 6	Step 7

Note: In the above steps, in case of difficulty, students will receive the guidance or support of teachers (especially for poor students).

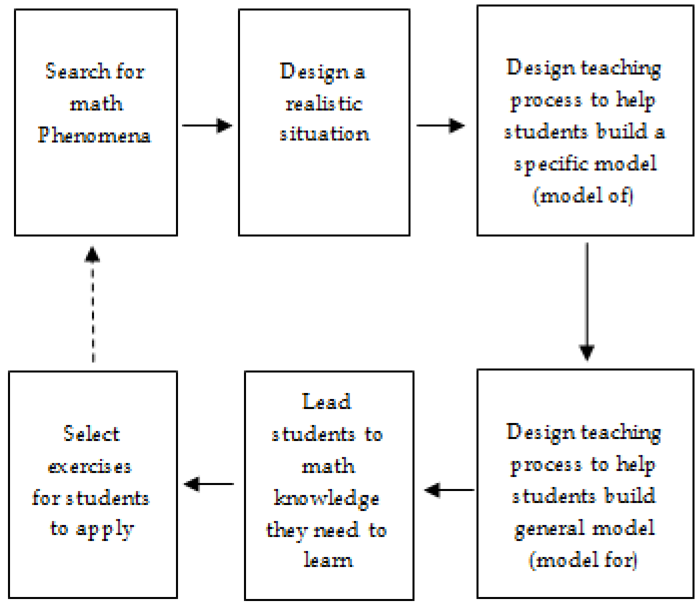

Figure 3: *How to design an RME - based lesson*

2.2 Research procedures

- Mathematics content for experimental teaching: Cosine theorem in a triangle – Geometry 10 (Vietnam)
- Classes used for experimental teaching

Class 10A4 (34 students) and Class 10A8 (38 students) of Thieu Van Choi High School, Ke Sach District, Soc Trang Province, Vietnam

The first experiment teaching was implemented in the academic year 2018-2019, at Class 10A4.

The second experiment teaching, we improved and revised the first lesson, and continued doing the second experimental teaching in the academic year 2018-2019, at Class 10A8.

- Teaching methods used for the experiment: Using the model of RME - based learning with a realistic situation as follows:

Huy's and Khai's house are on a hillside. By direct measurement, the Huy's and Khai's houses were measured 300m apart; the distance between Khai's house and a "deserted" house near the foot of the hill is 900m. The directions from Khai's house to Huy's house and from Khai'shouse to "deserted" house form an 80-degree angle. Due to the complicated terrain, one could not measure the distance directly from the Huy's house to the deserted house. So, is there any other way to calculate the distance from Huy's house to the deserted house? (see Figure 4)

- Questions (tasks) for students in the teaching process (see Table 2).

Question 1, 2, 3, 4 used for both Class10A4 and Class 10A8. Question 5 used for Class 10A4 is different from the one used for 10A8

Figure 4: *Context for teaching Cosine theorem*

Table 2: *Questions (for students) used in the teaching process*

Question	Objective
1. Transfer the practical problem to problem on a triangle and draw illustrations?	Build a specific model of the problem (model of)
2. Find the vector relationships related to three vertices of the triangle?	Recall known knowledge of vector in order to find strategies for solving the problem
3. Which relationship do you use to solve the problem?	Determine a strategy for solving the problem
4. How to apply the formula in question 3 to calculate the distance required by math problem?	Present the solution to the problem
5 (for Class 104). a. What is the problem that has just been solved in general form? b. Find relationships among three side of a triangle	Find the general model (model for) of the problem
5 (for Class 10A8). From the above problem, please fill in the blank below: a. The general form of the above problem: In any triangle, given ...... and the angle..... Calculate…. b. Results of the general problem can be stated as follows: In any triangle, one side is equal to ………… the remaining two sides, minus …………… of them and the cosine of the angle ………….	Find the general model (model for) of the problem and the content of Cosine theorem
6 (for Class 10 A8). In any ABC triangle, AB = c, AC = b, BC = a. Which formula for calculating a^2, b^2, c^2	Write down answers, which are contents of Cosine theorem.

2.3 Data Analysis

Research data were analyzed by descriptive statistics, and from students' answers recorded on the learning sheets, we used qualitative analysis to make comments and conclusions.

3.RESULTS

Students' the results of answering were shown in Table 3.

Table 3: *Statistics of results of students 'answers*

Question	The results of student 10A4			The results of student 10A8		
	Correct	False	No answer	Correct	False	No answer
1. Transfer the practical problem to problem on a triangle and draw illustrations?	18 52,9 %	14 41,2 %	2 5,9%	33 86,8 %	5 13,2 %	0 0%
2. Find the vector relationships related to three sides of the triangle?	29 85,3 %	4 11,8 %	1 2,9%	35 92%	3 7,8%	0 0%
3. Which relationship do you use to solve the problem?	22 64,7 %	12 35,3 %	0 0%	38 100%	0 0%	0 0%
4. How to apply the formula in question 3 to calculate the distance required by math problem?	13 38,2 %	17 50%	4 11,8 %	34 89,5 %	4 10,5 %	0 0%
5 (for Class 104) a. What is the problem that has just been solved in general form? b. Find relationships among three side of a triangle	8 23,5 %	22 64,7 %	2 5,9%			
5 (for Class 10A8). From the above problem, please fill in the blank below: a. The general form of the above problem: In any triangle, given ………… and the angle …………Calculate ……..? b. Results of the general problem: In any triangle, one side is equal to ………… the remaining two edges, minus …………… of them and the cosine of the angle ………….				37 97,4 %	1 2,6%	0 0%
6 (for Class 10 A8). In any ABC triangle, AB = c, AC = b, BC = a. Which formula for calculating a^2, b^2,				38 100%		0 0%

c^2						

4 DISCUSSION

About Question 1
In the first experiment (at Class10A4), teachers did not analyze the real situation in depth because we wanted to test the ability to read and understand the problem, as well as the ability to transform practical situations into mathematical problems to solve. Data show that about 50% of students understood the question and the answers met the question's request, about 41% of the students did not respond correctly to the request; the reason was that students did not understand what mathematization was or in other words, they were not clear about the requirements of turning practical situations into mathematical problems. In the second experiment (at Class 10A8), teachers explained more about mathematization and gave more guidance on practical situations. The number of students who correctly answered the questions increased significantly, about 86%.

About Question 2
In this question, both experimental classes answered quite well. 85% of students of class 10A4 answer correctly as required, and in experimental class 10A8 2, 92% of students answered the questions correctly; i.e., they listed some vector relationships related to the elements of a triangle. Most students did well on this question; it was apparent because they have just learned vector topics in previous lessons.

About Question 3
In the Class 10A4, there were a number of students wrote out the vector relationship related to the centroid of the triangle ($\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$), it was not helpful. For Class 10A8, teachers enhanced guidance's; therefore, students showed relationships related to the three sides of the triangle, such as the three-point rule, vector - addition rule, vector- subtraction rule. For this question, there were two options that were most popular in both classes: vector- addition rule and vector-subtraction rule. The teacher explained the reason for not choosing vector- addition rule to solve the problem: to solve this problem; we could also use vector- addition rule; however, if we used vector-addition rule, the position of the angle between the two vectors changed; so, the process of proving and calculating would take longer, we needed to choose the most suitable relationship, which was the subtraction rule of the two vectors to solve the problem.

About Question 4
It is possible to use various vector equalities involving the three vertices of a triangle to solve the given problem. However, using $\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}$ (1) would be the most convenient way in this case. To avoid wasting time for students, the teacher recommended that students should use formula (1). Thanks to the teacher's suggestion, about 38% of students in Class 10A4 gave correct answers, the remaining students could not give the final result because they did not remember the scalar product of two vectors. In Class 10A8, under the much detail guidance of the teacher, the number of students calculating the edge length was larger than the number of students in Class 10A4 and about 89.5% of students gave the correct results.

About Question 5 (10A4)
Because of the high generalization of the questions (such questions have never been done in previous assignments that students have done), and moreover, we allowed students of Class 10A4 to do the exercises themselves without with the guidance of teachers, only 24 students (70.6%) were able to state the given problem in a general form correctly. To overcome this limitation, in the Class, 10A8, we changed question 5 (see Table 2). As a result, almost all students in the Class 10A8 correctly completed this question

About Question 5 (10A8)
These questions are the type of filling in the blanks; the questions asked students to generalize the problem they just solved. The reason that we used these types of the question was to provide partial support for students to make generalizations; when doing the test, students only needed to select the appropriate words to fill in the blanks, it would be much easier if they asked them to state the full content . As a result, there were 37 (97.4%) out of 38 students who filled in the correct answer. It proved that this question was effective and made students closer to the more general formula.

About Question 6 (10A8)
From the answers to question 5 and the specific way of asking in question 6 (in addition, the teacher further explained the requirements of question 6), students in Class 10A8 wrote down the content of the cosine theorem easily and accurately.

CONCLUSION

In the research that we have done, the process of designing math lessons using the RME approach and the model of RME-based teaching have been verified quite successfully. With the teacher's guidance, students almost reinvented the cosine theorem. From the experimental teaching, we find that in order for RME - based teaching to be effective, teachers must be well prepared. First, it is crucial to choose a practical situation that is both interesting and able to derive the mathematical knowledge that the student needs to learn. Next, teachers need to prepare a system of guiding questions to help students build "model of" and "model for" step by step. Finally, the teacher needs to formally introduce the mathematical knowledge that student obtains after the course of the learning activities. Although our research was the case study, with positive results, we believe that RME-based teaching can fully meet the goal of enhancing the practicality of teaching mathematics in the coming period of Vietnam.

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