Fuzzy Version Of The Jalan-Ravallion Approach: Evidence From Tunisia

Amal Jmaii, Besma Belhadj

Abstract: Based on the recent literature, the present paper offers a new methodology to analyze poverty dynamics components. Building on the fuzzy subset theory, the aim of this study is to make available a new replay to the traditional Jalan-Ravallion approach by proposing new measures for chronic, transient and persistent poverty and provide new measure of observed poverty for each attribute. This paper proposes a new method using a membership function and fuzzy rules to identify to what extent individuals can be considered as poor/non-poor. According to the proposed membership function, a relative fuzzy index are adopted to compute an aggregation method of fuzzy individual poverty. The empirical assessment has been conducted using data of the Tunisian household survey from 2005 to 2010 and has been given arise to some potential policy recommendations through the fuzzy estimation results.

Index Terms: Chronic poverty, Fuzzy set approach, Membership function, Transient poverty, Tunisia

1 INTRODUCTION

For its socio-economic challenges, poverty is considered as a serious concern of public authorities. The emphasis on poverty reduction as a priority public policy requires the assessment of the evolution of this phenomenon over time [1], [12]. Nevertheless, the difficulty to conduct risk analyzes confront poverty, in developing countries, is due to the scarcity of panel data. Some authors have tried to overcome this difficulty through a cross-sectional analysis. Gibson [12] measure chronic poverty in Papua New Guinea, without a panel data, using a decomposed cross-sectional poverty method. Mehta and Shah [17], Okidi et al. [19] and many other authors determine chronic poverty based on household surveys. They have used information about household living standard to distinguish between chronic and transient poverty. Several economists, such as Gibson [12] and Suryahadi and Sumarto [22], used different methods to estimate the dynamic aspect of poverty without recourse to panel data. The tough challenge is then to identify poverty's dynamic components as accurately as possible taken into consideration the intrinsic nature of poverty. As a fuzzy measure, poverty is viewed as a degree and required two aspects in the analysis compared to the classic poor/non-poor dichotomous ranking. First, we distinguish a quantitative specification of individuals' degree of deprivation, by choosing the appropriate membership function. Second, the choice of axioms and rules for manipulating fuzzy sets such as union, intersections, complements and averaging. The purpose of our work is to propose a new approach that gives a nonparametric outlook to the application of fuzzy approaches to poverty dynamic analysis based on the Jalan-Ravallion approach. Panel data are very useful for studying the dynamics of individual's welfare.

- *Dr. Amal Jmaii, SOIE, Doctor of economic sciences (Specialty Quantitative Methods), Faculty of economic sciences and management of Tunis, Tunisia. E-mail: jemaiamal@yahoo.fr*
- *Dr. Besma Belhadj, SOIE, Associate professor, Faculty of economic sciences and management of Nabeul, Tunisia. E-mail: besma.kaabi@isg.rnu.tn*

In our case, where information is incomplete through time and surveys are not in form of panel data, we use the fuzzy sets approach. Betti et al. [6] and Wilson [19] have used this approach for a longitudinal study; therefore, they proposed a general rule to measure dynamic poverty using cross section and fuzzy operation such as the intersection as well as the union and the average. Betti and Verma [5], This paper presents a contribution to the extant literature about poverty dynamic studies in the case of independent surveys and suggests new statistical strategies to measuring poverty and welfare dynamics in Tunisia. Therefore, we compute fuzzy measures for chronic, transient and persistent poverty by proposing membership functions for each status of poverty on the base of the FGT index [8]. The remainder of this paper is divided as follow. In Section two, we present the proposed methodology. Results and interpretation are given in section three. Finally section four, concludes.

2 METHODOLOGY

2.1 Transient and chronic poverty according to the Jalan-Ravallion's approach

Jalan and Ravallion [10] proposed a methodology enables to decompose poverty into two components: chronic poverty, which reflecting the persistence of the phenomenon, and transient poverty, identifying conjectural aspects. Based on the FGT index ([8]), they define total poverty as follow:

$$P^{\alpha}(g_i) = \frac{1}{nt}\sum_{i=1}^{n}\sum_{j=1}^{t}(g_{ij})^{\alpha} \qquad (1)$$

With $g_{ij} = \max\left[\left(1 - y_{ij}\right), 0\right]$.

When $\alpha = 0$ we obtain the proportion of poor in the total population (n) at period (t). In the case when $\alpha = 1$, we obtain the average poverty gap and when $\alpha \succ 1$ we find the poverty severity index.

Chronic Poverty Measure:

Chronic poverty is defined as extreme poverty, which extends over a long period of time (several years) [10]. Over this period, individual who suffer from chronic poverty can not satisfy its minimum needs of food, clothing or housing. Based

on the FGT ([8]) class of poverty, chronic poverty can be defined as follows:

$$P_\alpha^c(Y) = \max\left(\frac{1}{t}\sum_{j=1}^{t}(1-\hat{\hat{y}}_i)^\alpha, 0\right) \qquad (2)$$

Where $P_\alpha^c(Y)$ is the chronic component of poverty and

$\hat{\hat{y}}_i = \frac{1}{t}\sum_{j=1}^{t} y_{ij}$ an estimation value of the permanent income during the t periods.

Transient poverty:
Transient poverty defines a situation where the individual has experienced at least one sequence of poverty and non-poverty on some periods of his life. Conventionally ([14], [15]), [16], [10], [23]) transient poverty represents the difference between total and chronic poverty.

$$P_\alpha^T(Y) = P^\alpha(g) - P_\alpha^c(Y) \qquad (3)$$

However, this classic poor/non-poor dichotomous ranking don't take into consideration the complexity of the poverty phenomenon. The tough challenge is then to identify poverty's dynamic components as accurately as possible taken into consideration the intrinsic nature of poverty. In this context fuzzy set logic, which define poverty as a membership degree, represent an alternative approach to the measurement of poverty.

2.2 Poverty components with the Fuzzy approach

Fuzzy set measurements require three steps: the proposing of the membership function, the manipulation of the fuzzy rules (axioms) and the computing of the poverty fuzzy index.

Membership function
The difference between crisp (i.e., classic) and fuzzy approach is established by introducing the concept of membership function. In contrast to crisp set, which requires only two alternatives poor/non-poor, in the fuzzy logic there are a whole continuum of truth value. As a result, fuzzy logic can be conceptualized as a generalization of crisp approach. Pursuing a methodology analogous to [4], [5], [2], [3] and [8] , we propose the following logic:

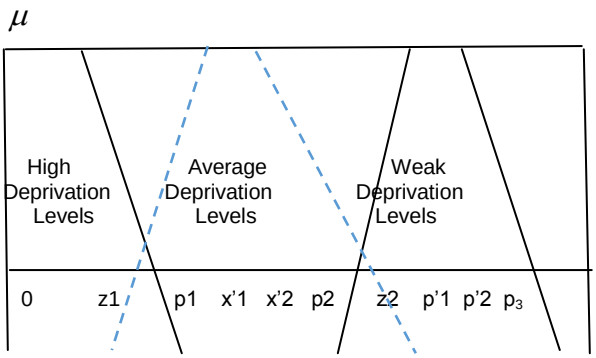

Fig. 1. *Different states of poverty*

We define a membership function which satisfy this logic as follow:

$$\mu^{HD}(x_i) = \begin{cases} 1 & if \ \ 0 \prec x_i \prec z_1 \\ \frac{-1}{P_1 - z_1}x_i + \frac{2P_1}{P_1 - z_1} & if \ \ z_1 \le x_i \prec P_1 \\ 0 & otherwise \end{cases} \qquad (4)$$

$$\mu^{AD}(x_i) = \begin{cases} \frac{6}{z_2 - z_1}x_i - 6\frac{z_1}{z_2 - z_1} & if \ \ z_1 \le x_i \prec x_1' \\ 1 & if \ \ x_1' \le x_i \prec x_2' \\ \frac{-1}{P_1 - z_1}x_i + \frac{2P_1}{P_1 - z_1} & if \ \ x_2' \le x_i \prec z_2 \\ 0 & otherwise \end{cases} \qquad (5)$$

$$\mu^{WD}(x_i) = \begin{cases} \mu_a^{WD} = \frac{6}{P_3 - P_2}x_i - 6\frac{P_2}{P_3 - P_2} & if \ \ P_2 \le x_i \prec P_1' \\ 1 & if \ \ P_1' \le x_i \prec P_2' \\ \mu_b^{WD} = \frac{-6}{P_3 - P_2}x_i + \frac{6P_3}{P_3 - P_2} & if \ \ P' \le x_i \prec P_3 \\ 0 & otherwise \end{cases} \qquad (6)$$

x_i is the i[th] household expenditure, P_1, P_2 and P_3 are respectively different values of expenditure that verify $\mu^{HD} = 0$, $\mu_a^{WD} = 0$ and $\mu_b^{WD} = 0$. Moreover, we define x_1', x_2', P_1' and P_2' are respectively expenditure values that verify $\mu_a^{AD} = 1$, $\mu_b^{AD} = 1$ and $\mu_b^{WD} = 1$, $\mu_b^{WD} = 1$.

Fuzzy rules combination
In this step, we propose a new fuzzy method in order to compute final fuzzy output for persistent poverty, chronic poverty and transitory poverty. We define three states of poverty: high deprivation, average deprivation and weak deprivation (figure 1). On the base of these states, we propose the following rules:

1. If the observed household has an income below a level z1, throughout the observed period, from poverty line and it belongs to the higher-level deprivation state with a higher probability. It is therefore considered as persistently poor. However, if his income exceeds the level z1, but still close he belongs to the average deprivation state with a low degree (figure 1).
2. In the case when the household has, an income between two limits z1 and z2 from the poverty line and he belongs to the average deprivation state with a degree very close to one. His is considered as chronically poor. This

household may also suffer from a transient poverty if his income exceeds the limit z2 but still close (i.e. he also belongs to the weak deprivation state with a low degree (figure 1)).

3. If, at each date of the observed period, the household has an income above the level Z 2 from poverty line but low than a certain value Z > Z 2 , then he has a low poverty with a very high degree of belonging to the week deprivation state. This poverty may be transient
4. If at each date of the observed period, the household has an income above the poverty threshold level, he will be considered as non-poor (i.e. he is outside the states of poverty)

Individual measure of chronic poverty and transient poverty

Note that μ^{HD}, μ^{AD} and μ^{WD} represent, respectively, the membership function of the subset HD, AD and WD and are individual measure of persistent, chronic and transient poverty. According to rule 1, we compute fuzzy persistent poverty of a household *i* in time *t* as follow:

$$\mu_i^{Pe} = \mu_{i1}^{HD} \Lambda \mu_{i2}^{HD} \qquad (7)$$

According to the same rule, this household may suffer from a chronic poverty if his income above Z_1 (but still close), and his membership function may be computed as follow:

$$(\mu_{i1}^{HD} \Lambda \overline{\mu}_{i2}^{HD}) \vee (\overline{\mu}_{i1}^{HD} \Lambda \mu_{i2}^{HD}) \qquad (8)$$

From rule 2, chronic poverty for a household $i \neq j$ is represented as follow:

$$\mu_i^{ch} = \vee_{t=1,\dots,T} \mu_{it}^{AD} \qquad (9)$$

According to both rule 1 and rule 2, an individual chronic poverty may be computed as:

$$\mu_{ij}^{ch} = (\mu_{i1}^{HD} \Lambda \overline{\mu}_{i2}^{HD}) \vee (\overline{\mu}_{i1}^{HD} \Lambda \mu_{i2}^{HD}) \vee (\mu_{j1}^{AD} \Lambda \overline{\mu}_{j2}^{AD}) \quad (10)$$

However, this household may suffer from a transient poverty if his income above Z_2 (but still close):

$$(\mu_{i1}^{AD} \Lambda \overline{\mu}_{i2}^{AD}) \vee (\overline{\mu}_{i1}^{AD} \Lambda \mu_{i2}^{AD}) \qquad (11)$$

According to rule 3, transient poverty may be computed as:

$$\mu_k^{Tr} = \Lambda_{t=1,\dots,T} \mu_{it}^{WD} \qquad (12)$$

Therefore, an individual transient poverty is presented as:

$$\mu_k^{Tr} = (\mu_{i1}^{AD} \Lambda \overline{\mu}_{i2}^{AD}) \vee (\overline{\mu}_{i1}^{AD} \Lambda \mu_{i2}^{AD}) \vee (\mu_{k1}^{WD} \wedge \mu_{k2}^{WD}) \quad (13)$$

Aggregate measurement of persistent, chronic and transient poverty

In this paragraph, we propose a fuzzy index enable to compute global measurement of persistent poverty, chronic poverty and transitory poverty. This index is equal to the weighted average individual measure of each poverty components. In fact, the weighting is the share of household equivalent size where s = I, j, or k

$$I_t^*(\theta) = \sum_{l=1}^{m} w_l \sum_{s=i,j,k} \frac{n_s^\theta}{n} \mu^*{}_t(x_s) \qquad (14)$$

With *= Tr, Ch or Pe and l = 1,…, m

θ and w_l are respectively size elasticity and the share of each attribute.

Observed poverty

According to the Jalan-Ravallion approach, observed poverty is equal to the sum of persistent poverty, chronic poverty and transitory poverty. Following this definition, we propose a fuzzy measure of observed poverty:

$$\mu_{it}^{o} = \mu_{it}^{tr} + \mu_{it}^{pe} + \mu_{it}^{ch} \qquad (15)$$

In addition, for an aggregate measure of observed poverty we propose the following index:

$$I_t^o(\theta) = \sum_{l=1}^{m} w_l \sum_{s=i,j,k} \frac{n_s^\theta}{n} \mu^o{}_t(x_s) \qquad (16)$$

3 DATA APPLICATION AND RESULTS

In this paragraph, we apply the proposed methodology of section 2, not only to compare different regions in Tunisia, but also to compare poverty in rural versus urban zone, according to educational level of the household head. The present empirical analysis has been conducted using the household survey data conducted by the INS in 2005 and 2010. The surveys provide demographic, economic and social characteristics of households. In order to take into account the different geographical and socioeconomic characteristics of the regions, in Tunisia, the country is divided into five different homogenous regions, two of which are urban areas. We have selected as attribute, the region of household residence: Great Tunisia, urban east, rural east, urban west and rural west. We choose this decomposition since we are interested by the urban rural disparities over time. In addition, we selected the educational level of household heads: Illiterate, primary level, secondary level and higher level. In this study, consumption expenditures are taken as indicators of welfare. A brief summary of this variables is given in Table 1.

TABLE I

Descriptive Statistics of the variable Total annual Household Expenditure (2005/2010)

Variables	mean	min	std	max
Total expenditures (2005)	1939806	36941.8	1808218	5400000
Total expenditures (2010)	2322616	57420.01	1969094	6000000

3.1 Fuzzy poverty components per region

In the present paragraph, the methodology described above is implemented within the five Tunisian regions. Results show that for both 2005 and 2010, rural west and rural east suffer from a persistent poverty. In contrast, Great Tunisia region and urban east suffer from a transient poverty (table2 and table3).

TABLE II

Fuzzy poverty by region – 2005

Region	[z1, z2]	ω_r^*	μ_t^{pe}	μ_t^{ch}	μ_t^{tr}	μ_t^{o}
Great Tunisia	[1032, 2157]	0.205	0.072	0.104	0.090	0.266
Urban west	[831, 1526]	0.163	0.098	0.156	0.016	0.270
Rural west	[661, 1211]	0.210	0.117	0.212	0.033	0.362
Urban east	[983, 1817]	0.279	0.054	0.073	0.106	0.233
Rural east	[713, 1412]	0.143	0.113	0.194	0.049	0.356
Global fuzzy poverty		**1**	**0.090**	**0.148**	**0.059**	**0.279**

TABLE III

Fuzzy poverty by region – 2010

Region	[z1, z2]	ω_r^*	μ_t^{pe}	μ_t^{ch}	μ_t^{tr}	μ_t^{o}
Great Tunisia	[1032, 2157]	0.205	0.072	0.104	0.090	0.266
Urban west	[831, 1526]	0.163	0.098	0.156	0.016	0.270
Rural west	[661, 1211]	0.210	0.117	0.212	0.033	0.362
Urban east	[983, 1817]	0.279	0.054	0.073	0.106	0.233
Rural east	[713, 1412]	0.143	0.113	0.194	0.049	0.356
Global fuzzy poverty		**1**	**0.090**	**0.148**	**0.059**	**0.279**

In addition, our finding emphasize, in contrast to the INS's results that great Tunisia is not the less vulnerable to poverty region; it is rather the urban east region.

3.2 Fuzzy poverty components per educational level

Educational has an important effect on household's well-being. Results show, that individuals who are illiterate or have just a primary level suffer from a higher level of transient poverty than other groups. These groups might suffer from periodical change in their status. Ceteris paribus, more we are graduated more we are able to improve our well-being. These find does not surprise us and it is in accordance with earlier work ([3]).

TABLE IV

Dynamic fuzzy poverty by educational level – 2005

Region	[z1, z2]	ω_r^*	μ_t^{pe}	μ_t^{ch}	μ_t^{tr}	μ_t^{o}
Illiterate	[661, 1211]	0.223	0.173	0.0168	0.119	0.460
primary level	[831, 1526]	0.390	0.114	0.125	0.098	0.337
Secondary level	[938, 1817]	0.299	0.097	0.113	0.055	0.265
High level	[1032, 2157]	0.087	0.053	0.078	0.019	0.150
Global fuzzy poverty		**1**	**0.109**	**0.121**	**0.072**	**0.302**

TABLE

Dynamic fuzzy poverty by educational level – 2010

Region	[z1, z2]	ω_r^*	μ_t^{pe}	μ_t^{ch}	μ_t^{tr}	μ_t^{o}
Illiterate	[661, 1211]	0.234	0.166	0.161	0.095	0.422
primary level	[831, 1526]	0.385	0.108	0.115	0.061	0.284
Secondary level	[938, 1817]	0.274	0.080	0.102	0.039	0.221
High level	[1032, 2157]	0.107	0.042	0.073	0.011	0.126
Global fuzzy poverty		**1**	**0.099**	**0.112**	**0.051**	**0.262**

4 Conclusion

In conclusion, the definition of a set of rule for following the household in a longitudinal way has allows us to be coherent with the concept of poverty in a dynamic context. The analysis of poverty in the long term is essential for the formulation of economic policies. This analysis allows us not only to identify the poor, but also to examine why some people do not get out from the poverty trap. Therefore, it allows decision makers to establish effective structural policies acting on the causes of the persistence of poverty situations in some households. In this paper, we propose a new approach for measuring poverty dynamics according to the fuzzy set reasoning. This works contribute to the debate of the fight against poverty and risk-vulnerability in Tunisia. However, it shows that the importance of distinguishing transient and chronic poverty depends on the nature of the used tools measurement. Contrary to Cheli (1995), fuzzy approach proves that chronic poverty is much higher than transient poverty. Most works on poverty dynamics, in developing country, consisted in measuring and identifying correlated variables to chronic and transient poverty. Further research are necessary to understand what causes chronic and persistent poverty and why some groups of individuals are note able to accumulate assets that generate sufficient income to increase consumption expenditures above

a minimum acceptable level.

REFERENCES

[1] M. J. Bane and D. T. Ellwood, “Slipping into and out of poverty: The dynamics of spells”, The Journal of Human Resources, vol.1, no 21, pp. 1–23, 1986.

[2] B. Belhadj, ‘’Seuil relatif, mesure floue de la pauvreté. Nouvelles propositions’’, Ethics and Economics, vol. 1, no 10, pp. 153-172. 2013.

[3] B. Belhadj, ‘’A new fuzzy poverty index by distinguishing three levels of poverty’’, Research in Economics, vol. 3 no 65, pp. 221-231. 2011.

[4] B. Cheli, ‘’Totally fuzzy and relative measures of poverty in dynamic context: An application to the British Household Panel Survey, 1991-1992’’, ESRC Research Centre on Micro-Social Change, Universisty of Essex, 1995.

[5] G. Betti, and V. K. Verma, ‘’Measuring the degree of poverty in a dynamic and comparative context: a multi-dimensional approach using fuzzy set theory’’, ICCS-VI , Vol. 11, p. 289. 1999.

[6] G. Betti, B. Cheli, and R. Cambini, ‘’A statistical model for the dynamics between two fuzzy states’’, theory and application to poverty analysis, Metron. vol 3, no 62, pp. 391-411. 2004.

[7] G. Betti, B. Cheli, and V. Verma, ‘’On longitudinal analysis of poverty conceptualized as a fuzzy state””. Working paper series, ECINEQ WP, vol.32. 2006.

[8] A. Chatterjee, S. Mukherjee, and S. Kar, ‘’Poverty level of households : A multidimensional approach based on fuzzy mathematics’’, Fuzzy Information and Engineering, vol.6 no. 4, pp. 463-487. 2014.

[9] S. Chaudhuri, Assessing vulnerability to poverty: concepts, empirical methods and illustrative examples. Department of Economics Columbia University, http://info. worldbank. org/etools/docs/library/97185 /Keny_0304/Ke_0304/vulnerabilityassessment. 2003.

[10] J. Duclos, Y. Araar and J. Giles, ‘’Chronic and transient poverty: Measurement and estimation, with evidence from China’’. Journal of Development Economics, vol. 91, no 2, pp. 266-277. 2010.

[11] J. Foster, J. Greer, and E. Thorbecke, ‘’A class of decomposable poverty measures’’, Journal of the Econometric Society, pp. 761–766. 1984.

[12] J. Gibson, ‘’Measuring chronic poverty without a panel’’. Journal of Development Economics, vol. 65 no 2, pp. 243–266. 2001.

[13] D. Hulme and A. McKay, ‘’Identifying and measuring chronic poverty: Beyond monetary measures’’, Chronic Poverty Research Centre Working Paper, no 30. 2005.

[14] J. Jalan and M. Ravallion , ‘’Transient poverty in postreform rural china’’, Journal of Comparative Economics, vol 26, no. 2, pp. 338–357. 1998.

[15] J. Jalan, and M.. Ravallion, ‘’Is transient poverty different? Evidence for rural China’’. Journal of Development Studies, 36(6), 82-99. 2000.

[16] T. Kurosaki, ‘’the measurement of transient poverty: Theory and application to Pakistan’’. Journal of Economic Inequality, vol. 4, no 3, pp. 325-345. 2006.

[17] A. K. Mehta and A. Shah, ‘’Chronic poverty in india: overview study’’. Chronic Poverty Research Centre Working Paper (7). 2001.

[18] A. Sen, ‘’Poverty: an ordinal approach to measurement”, Journal of the Econometric Society, pp. 219–231. 1976

[19] , J. Okidi, G. Bahiigwa, and G. Kempaka, ‘’Costing the Millennium Development Goals: Uganda Country Study’’. Economic Policy Research Centre. 2002.

[20] A. H. Stevens, ‘’the dynamics of poverty spells: updating bane and ellwood’’, The American Economic Review, pp.34–37. 1994.

[21] A. H. Stevens, ‘’Measuring the persistence of poverty over multiple spells: Climbing out of poverty falling back’’, Journal of Human Resources, vol. 34, no.3, pp. 557–588. 1999.

[22] A. Suryahadi, and S. Sumarto, ‘’Poverty and vulnerability in Indonesia before and after the economic crisis’’. Asian Economic Journal, vol. 17, no 1, pp. 45–64. 2003.

[23] G. Wan and Y. Zhang, ‘’Chronic and transient poverty in rural China’’. Economics Letters, vol. 119, no 3, pp. 284-286. 2013.