Intuitionistic Fuzzy Multiset (IFMS) In Binomial Distributions

P. A. Ejegwa, J.A. Awolola

Abstract: We proposed the application of IFMS in binomial distributions. This is possible as we assumed theoretically that the probability of the membership degrees is constant for each of the trials and that the intuitionistic fuzzy multiset index is negligible i.e. $\pi \approx 0$. We made used of the Bernouilli trials formula to find the binomial probability of intuitionistic fuzzy multisets.

Keywords: binomial probability, fuzzy sets, intuitionistic fuzzy sets, fuzzy multisets, intuitionistic Fuzzy multisets.

1. Introduction

Modern Set theory formulated (or invented) by a German mathematician George Cantor (1845-1918) is fundamental and indispensable for the whole of Mathematics. In fact, Set Theory (ST) is the language of Science, Mathematics, Logic and Philosophy. However, one cardinal issue associated with the notion of a set is the concept of vagueness. Obviously, every one believes that, in accordance to the tenet of Mathematics, all mathematical notions including set must be exact. This vagueness or the representation of imperfect knowledge has been a challenge for a long time for philosophers, logicians, scientists and mathematicians. Notwithstanding, of recent it became a crucial issue for computer scientists, particularly in the area of artificial intelligence. To handle situation like this, many tools were suggested viz; fuzzy sets, multisets, rough sets, soft sets, fuzzy multisets and intuitionistic fuzzy sets [8]. Considering the unpredictable factors in decision-making, Zadeh [1] introduced the idea of fuzzy set which has a membership function that assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of belongingness to the set under consideration. Atanassov [2] subsequently proposed the concept of intuitionistic fuzzy set (IFS) by bringing a non-membership function together with the membership function of the fuzzy set introduced earlier by Zadeh [1]. Among the various notions of higher-order fuzzy sets, IFS proposed by Atanassov[4] provides a flexible framework to elaborate uncertainty and vagueness. This idea of IFS seems to be resourceful in modeling many real life situations like negotiation processes, psychological investigations, reasoning, medical diagnosis among others [8]. Nevertheless, it is worthy to note that many field of modern Mathematics have been emerged by violating a basic principle of a given theory only because useful structures could be defined this way. For instance, set which is a well-defined collection of distinct and definite objects, that is, the elements of a set are pair-wise different. If wesnub this restriction and allow repeated occurrences of element, then we can get a mathematical structure called bags or lists or multisets. For example, the prime factorization of an integer $n > 0$ is a multiset whose elements are primes. The number *240* has the prime factorization $240 = 2^4 3^1 5^1$ which gives the multiset $\{2,2,2,2,3,5\}$[6, 7]. As a generalization of fuzzy sets, Yager [5] introduced the concept of fuzzy multiset (FMS). An element of a fuzzy multiset can occur more than once with possibly the same or different membership values. Then years after, Shinoj and Sunil [8]made an attempt to combine the two concepts: IFS and FMS together by introducing a new concept called intuitionistic fuzzy multiset. In this paper, we propose the application of intuitionistic fuzzy multisets in binomial distributions. This application can be seem if we assume theoretically that the probability of the membership degree is the same for each of the trials and that the hesitation margin is very insignificant.

- *P. A. Ejegwa, J.A. Awolola*
- *Department of Mathematics/Statistics/Computer Science*
- *University of Agriculture, P.M.B. 2373, Makurdi-Nigeria*

2. Preliminaries

Definition 1: Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$, where $\mu_A(x): X \longrightarrow [0,1]$ is the membership function of the fuzzy set A.

Definition 2: Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x): X \longrightarrow [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of non-determinacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e., $\pi_A(x): X \longrightarrow [0,1]$ for every $x \in X$. $\pi_A(x)$, expresses the lack of knowledge of whether x belongs to IFS A or not.

Definition 3: Let X be a nonempty set. A fuzzy multiset (FMS) A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A s.t. $CM_A: X \longrightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval [*0,1*]. Then for any $x \in X$, the value $CM_A(x)$ is a crisp multiset drawn from [*0,1*]. For each $x \in X$, the membership degree is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^n(x))$, where $\mu_A^1(x) \geq \mu_A^2(x) \geq \ldots \geq \mu_A^n(x)$.

3. Concept of Intuitionistic Fuzzy Multisets (IFMSs)

Definition4: Let X be a nonempty set. An IFMS A drawn from X is characterized by two functions: "count membership" of A denoted as CM_A and "count non-membership" of A denoted as CN_A given respectively by $CM_A: X \longrightarrow Q$ and $CN_A: X \longrightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval *[0,1]* s.t. for each $x \in X$, the membership degrees of element in $CM_A(x)$ is defined as a decreasingly ordered sequence and it is denoted as $(\mu_A^1(x), \mu_A^2(x), ..., \mu_A^n(x))$, where $\mu_A^1(x) \geq \mu_A^2(x) \geq ... \geq \mu_A^n(x)$ whereas the corresponding non-membership degrees of element in $CN_A(x)$ is denoted by $(\nu_A^1(x), \nu_A^2(x), ..., \nu_A^n(x))$ *s.t.* $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ for every $x \in X$ and $i = 1, . . . , n$. This means, an IFMS A is defined as; $A = \{\langle x, CM_A(x), CN_A(x)\rangle : x \in X\}$ or $A = \{\langle x, \mu_A^i(x), \nu_A^i(x)\rangle : x \in X\}$, *for* $i = 1, . . . , n$. For each IFMS A in X, $\pi_A^i(x) = 1 - \mu_A^i(x) - \nu_A^i(x)$ is the intuitionistic fuzzy multisets index or hesitation margin of *x* in A. The hesitation margin $\pi_A^i(x)$ *for each* $i = 1, . . . , n$ is the degree of non-determinacy of $x \in X$, to the set A and $\pi_A^i(x) \in [0,1]$.Thefunction $\pi_A^i(x)$expresses lack ofknowledgeof whether x *in* X or not. In general, an IFMS A is given as $A = \{\langle x, \mu_A^i(x), \nu_A^i(x), \pi_A^i(x)\rangle : x \in X\}$.

Note: $\mu_A^i(x) + \nu_A^i(x) + \pi_A^i(x) = 1$.

4. Operations in Intuitionistic Fuzzy Multisets

For any two IFMSs A and B drawn from X, the following operations hold.

[Complement] $A^c = \{\langle x, \nu_A^i(x), \mu_A^i(x)\rangle : x \in X\}$

[Union] $A \cup B = \{\langle x, \max(\mu_A^i(x), \mu_B^i(x)), \min(\nu_A^i(x), \nu_B^i(x))\rangle : x \in X\}$

[Intersection] $A \cap B = \{\langle x, \min(\mu_A^i(x), \mu_B^i(x)), \max(\nu_A^i(x), \nu_B^i(x))\rangle : x \in X\}$

[Addition] $A \oplus B = \{\langle x, (\mu_A^i(x) + \mu_B^i(x) - \mu_A^i(x)\mu_B^i(x)), (\nu_A^i(x)\, \nu_B^i(x))\rangle : x \in X\}$

[Multiplication] $A \otimes B = \{\langle x, (\mu_A^i(x)\mu_B^i(x)), (\nu_A^i(x) + \nu_B^i(x) - \nu_A^i(x)\, \nu_B^i(x))\rangle : x \in X\}$.

5. Cardinality of Intuitionistic Fuzzy Multisets

Definition 5: The length of an element x in an IFMS A is defined as the cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 \leq \mu_A^i(x) + \nu_A^i(x) \leq 1$ and it is denoted by $L(x: A)$ i.e. the length of x in A for each $x \in X$. Then, $L(x: A) = |CM_A(x)| = |CN_A(x)|$.

Definition 6: If A and B are IFMSs drawn from X, then $L(x: A, B) = \text{Max}\{L(x: A), L(x: B)\}$ or $L(x) = V[L(x: A), L(x: B)]$ where $L(x) = L(x: A, B)$ and V denotes maximum.

Note:

1. In an IFMS, $|\mu_A^i(x)| = |\nu_A^i(x)|$ for each $i = 1,2, ..., n$.
2. Whenever $i = 1$, an IFMS becomes IFS.
3. IFMS and FMS of the same length have equal cardinality.

Forexample, for the set $X = \{x\}$ with $A = \{\langle(0.3, 0.2), (0.4, 0.6)\rangle\}$ and $B = \{\langle(0.5, 0.4, 0.3, 0.2), (0.4, 0.6, 0.6, 0.7)\rangle\}$. Then $L(x: A) = 2$, $L(x: B) = 4$ and $L(x) = 4$.

Definition 7(Simple IFMS): Let X be nonempty. An IFMS A drawn from X is said to be simple if all its membership degrees are the same or have the same grade i.e. $\mu_A^1(x) = \mu_A^2(x) = \cdots = \mu_A^n(x)$. Since $\mu_A^i(x)$ and $\nu_A^i(x)$ are complementary as $\pi_A^i(x) = 0$, it then implies that $\nu_A^1(x) = \nu_A^2(x) = \cdots = \nu_A^n(x)$. For example, $A = \{\langle(0.4, 0.4, 0.4), (0.6, 0.6, 0.6)\rangle\}$.

Definition 8(Multiple of IFMS): Let X be nonempty. The multiplicity or frequency of an IFMS A is the number of time the membership degree (with the non-membership degree) appears. For example, $A = \{\langle(0.3, 0.4, 0.0), (0.7, 0.6, 1.0)\rangle\}$ has the multiplicity of *3* i.e. $i = 1, 2, 3$ where i is the step.

6. Application of Intuitionistic Fuzzy Multisets in Binomial Distribution

The binomial distribution arises from a repeated random experiment which has two possible outcomes:

1. that an event will occur i.e. degree of membership μ.
2. that an event will not occur i.e. degree of non-membership ν.

For example, in the experiment of tossing a fair coin repeatedly, there are two possible outcomes namely; a head or not a head (i.e. membership degree or non-membership degree). For each repetition of tossing of the fair coin is called a trial. The two possible outcomes of the random experiment are usually called success and failure. If the probability that a membership degree μ shows up is a success, the probability that a non-membership degree ν shows up is a failure, and conversely. Since membership degree and non-membership degree are complementary, it implies that $\mu + \nu = 1$ or $\nu = 1 - \mu$ i.e. $\pi = 0$.

Note: 1. In the binomial distribution, the hesitation margin is negligible i.e. $\pi = 0$ as in normalization of IFS [9].

2. We assume theoretically that the probability of membership degree is the same for each of the trials i.e. simple IFMS.

The independent trials discussed above are often called Bernoulli trials. If we denote the probability that in n-Bernouilli trails, we have i successes and $n - i$ failures by $f(i)$, then $f(i)$ is defined as thus: $f(i) = C(n, i)\mu^i \nu^{n-i}$, where $C(n, i) = \frac{n!}{(n-i)!i!}$ and n is the number of trials.

Note: $f(i)$ is called binomial probability (or frequency or density function) or binomial distribution [3].

Example

Given that $A = \{\langle(0.3, 0.3, 0.3, 0.3), (0.7, 0.7, 0.7, 0.7)\rangle\}$, the binomial probability of A in at least step *2* is thus: A has the multiplicity of *4* i.e. $n = 4$ and *4* steps since $i = 1,2,3,4$. In at least step *2* means in step *2,3* and *4*. $f(i) = C(n,i)\mu^i \nu^{n-i}$, $\mu = 0.3$ and $\nu = 0.7$.

$$f(\,at\ least\ in\ step\ 2) = f(2) + f(3) + f(4).$$

$$f(2) = C(4,2)0.3^2 0.7^2 = 0.2646$$

$$f(3) = C(4,3)0.3^3 0.7^1 = 0.0756$$

$$f(4) = C(4,4)0.3^4 0.7^0 = 0.0081$$

Therefore, the required binomial probability of A in at least step *2* is *0.3483*.

7. Conclusion

The notion of intuitionistic fuzzy multiset is useful in binomial distributions whenever the intuitionistic fuzzy multiset index is negligible.

References

[1]. L.A. Zadeh, Fuzzy sets, Inform.and Control 8 (1965) 338-353.

[2]. K. Atanassov, Intuitionistic fuzzy sets, VII ITKR's Session, Sofia, 1983.

[3]. M. L. Boas, Mathematical methods in physical sciences, Second edition, Libraryof Congress Cataloging in publication data (1983) 718-720.

[4]. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.

[5]. R. R. Yager, On the theory of bags, Int. J. of General Systems 13 (1986) 23-37.

[6]. W.D. Blizzard, Multiset theory, Notre Dame Journal of formal logic 30 (1989).

[7]. W.D. Blizzard, Dedekind multisets and function shell, Theoretical Computer science 110 (1993) 79-98.

[8]. T.K. Shinoj, J. J. Sunil, Intuitionistic fuzzy multisets and its application in medical diagnosis, Int. J. of Mathematical and Computational Sciences 6 (2012).

[9]. A.M. Ibrahim, P.A. Ejegwa, Remark of some operations on Intuitionistic fuzzy sets, Int. Journal of Science and Technology Vol. 2,1 (2013) 94-98.