Five Dimensional String Cosmological Model

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Abstract: - In this paper, it is observed that five dimensional Kaluza-Klein space-time in the presence of cosmic string in the frame work of Seaz and Ballester scalar tensor theory of gravitation, the string cosmological models corresponding to geometric string, Takabayasi string and Reddy string are same. Also some physical and kinematical properties of the models are discussed.

Keywords: - viscous fluid, Five dimensional cosmological models, string.

1 INTRODUCTION

The higher dimensional cosmological models play an important role in the description of the universe in its early stages of evolution. Therefore the study of higher dimensional cosmological models in recent years is highly motivated the researchers mainly by the possibility of geometrical unification of the fundamental interaction of the universe. Five dimensional space-time is more attractive because 10D and 11D super gravities admit solutions, which spontaneously reduces to 5D. In these days many efforts have been made to construct alternative theories of gravitation. Einstein's idea of geometrizing gravitation in general theory of relativity motivated others to geometrize other physical things. Exact solutions of Einstein's field equation in higher dimensions are of great interest in several contexts in view of the modern Kaluza-Klein theory. The extra dimensions play a physical role and they are being too small are unobservable. Chodos and Detweiler (1980) constructed a cosmological model which shows the contraction of the extra dimensions as a consequence of cosmological evolutions Guth (1981), Alvarez and Gavela (1983) observed that during contraction process extra dimensions produce large amount of entropy, which provides an alternative resolution to the flatness and horizon problems as compared to usual inflationary scenario. Further Freund (1982), Appelquist and Chodos (1983), Randjbar-Daemi et al. (1984), Rahaman et al. (2002), Singh et al. (2004) and Mohanty et al. (2006) claimed through solutions of the field equations that there is an expansion of four dimensional space-time while fifth dimension contracts or remains constant. Very recently the study of string cosmological models in alternative theory of gravitation is gaining momentum. Sen (2000), Barros et al. (2001), Sen et al. (1997), Gundalach and Ortiz (1990), Barros and Romero (1995), Bhattacharjee and Baruah (2001), Rehaman et al. (2003) and Reddy (2005) have presented string cosmological models in alternative theories of gravitation. In particular, Reddy (2003a, 2006) and Reddy et al. (2006) have discussed some string cosmological models in Saez-Ballester scalar-tensor theory of gravitation in four dimensions. In this paper, we have discussed Nambu-Takabayasi and Reddy's string cosmological models in five dimensional space-time in the frame work of scalar-tensor theory of gravitation.

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2 METRIC AND FIELD EQUATIONS

Consider the five dimensional Kaluza-Klein metric in the form

$$ds^2 = -dt^2 + R^2(t)\left(\frac{dr^2}{1-k\,r^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right) + A^2(t)d\square^2 . \qquad (1)$$

Unlike Wesson (1983), the fifth co-ordinate is taken to be space-like and the metric coefficients are assumed to be functions of time only. Here the spatial curvature has been taken as zero (Gron 1988). The field equations given by Seaz and Ballester (1985) for the combined scalar and tensor fields are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = T_{ij} \qquad (2)$$

and the scalar field satisfies the equation

$$2\phi^n\phi^i_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 , \qquad (3)$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$, is the Einstein tensor, ω and n are constants, T_{ij} is the energy tensor of the matter. Comma and semicolon denotes partial and covariant differentiation respectively.

Also

$$T^{ij}_{;j} = 0 , \qquad (4)$$

is a consequence of the field equations (2) and (3).

The energy-momentum tensor for cosmic strings is,

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j . \qquad (5)$$

Here ρ is the rest energy density of the system of strings with massive particles attached to the strings and λ the tension density of the system of strings. As pointed out by Letelier (1983), λ may be positive or negative, u^i describes the system four velocities and x^i represents direction of anisotropy, i.e.

the direction of strings which is taken to be along fifth dimension.

We have

$$u^i u_i = 1,\ \ x^i x_i = -1 \quad \text{and} \quad u^i x_i = 0\ . \tag{6}$$

We consider, $\rho = \rho_p + \lambda$ (7)

where ρ_p is the rest energy density of particles attached to the strings. Here, we consider ϕ, ρ and λ are functions of *t* only.

The field equations (2), (3), and (4) for the metric (1) with the help of (5) and (6) can, explicitly, be written as

$$\frac{3R'^2}{R^2} + \frac{3R'A'}{RA} + \frac{3k}{R^2} + \frac{\omega}{2}\phi^n \phi'^2 = \rho\ , \tag{8}$$

$$\frac{2R''}{R} + \frac{R'^2}{R^2} + \frac{2R'A'}{RA} + \frac{k}{R^2} + \frac{A''}{A} - \frac{\omega}{2}\phi^n \phi'^2 = 0\ , \tag{9}$$

$$\frac{3R''}{R} + \frac{3R'^2}{R^2} + \frac{3k}{R^2} - \frac{\omega}{2}\phi^n \phi'^2 = \lambda\ , \tag{10}$$

$$\phi'' + \phi'\left(\frac{3R'}{R} + \frac{A'}{A}\right) - \frac{n}{2}\frac{\phi'^2}{\phi} = 0 \quad \text{and} \tag{11}$$

$$\rho' + (\rho - \lambda)\frac{A'}{A} + 3\rho\frac{R'}{R} = 0\ . \tag{12}$$

3 SOLUTIONS AND THE MODELS

The field equation (8) to (12) are four independent equations in five unknowns *R, A, ϕ, ρ and λ*. Hence to get a determinate solution one has to assume a physical or mathematical condition. In the literature, we have equations of state for string model (Letelier 1983), are

$\rho = \lambda$ (geometric string or Nambu string)

$\rho = (1 + \omega)\lambda$ (p-string or Takabayasi string)

In addition to above, recently, Reddy (2003a, 2003b), Reddy and Rao (2006); Reddy and Naidu (2007) have obtained inflationary string cosmological models in Brans and Dicke (1961), Seaz and Ballester (1985) and Lyra (1951) scalar-tensor theories of gravitation assuming a relation,

$\rho + \lambda = 0$ (Reddy string). (13)

i.e. the sum of rest energy density and tension density for a cloud of strings vanishes. The relation (13) is analogous to $\rho + p = 0$ in general relativity with perfect fluid as source which represents false vacuum case.

Here we find string cosmological models corresponding to
(i) $\rho = \lambda$,

(ii) $\rho = (1 + \omega)\lambda$ and

(iii) $\rho + \lambda = 0$ in five dimensions in Saez-Ballester scalar-tensor theory.

Case(i): Geometric string ($\rho = \lambda$)
Here we also assume $A = R^n$ because of the fact that field equations are highly non-linear. Using this relation, the field equation

(8 - 12) admits the exact solution

$$R = L_1 t + L_2 = T\ , \tag{14}$$

$$\rho = \lambda = \frac{(-n^2 - n + 2){L_1}^2 + 2k}{T^2}\ , \tag{15}$$

$$\phi = [\frac{(n-2)L_3}{2(n+2)L_1} T^{-(2+n)} + L_4]^{\frac{2}{-n+2}}\ . \tag{16}$$

After a suitable choice of co-ordinate and constants of integration, the model corresponding to the solution (14, 15, 16) can be written as

$$ds^2 = -dt^2 + T^2\left(\frac{dr^2}{1 - k\,r^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\right) + T^{2n} d?^2, \tag{17}$$

Spatial volume (V^3) : $V^3 = \sqrt{-g} = \frac{T^{3+n} r^2 \sin\theta}{\sqrt{1-kr^2}}$,

Scalar expansion *(θ)*: $\theta = \frac{1}{3}u^i_{;i} = \frac{1}{3T}$,

Deceleration parameter *(q)*: $q = -\frac{3}{\theta^2}\left[\theta_{;\alpha}u^\alpha + \frac{\theta^2}{3}\right] = 2 > 0$,

Shear Scalar (σ^2): $\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{1}{54T^2}$

Volume increases with time. Since *q* > 0, therefore the model is not inflationary. Shear decreases as time increases. Since, $\lim_{T\to\infty}\frac{\sigma}{\theta} \neq 0$, therefore the model is anisotropic for values of *T.*

In Case (ii): p-string or Takabayasi string ($\rho = (1 + \omega)\lambda$) and **Case (iii):** Reddy string ($\rho + \lambda = 0$).

It is observed that the five dimensional model in **Case (ii)** and **Case (iii)** remain same as in **Case (i).**

4 CONCLUSIONS

For dimensional Kaluza-Klein space-time in the presence of cosmic string in the frame work of Seaz and Ballester scalar tensor theory of gravitation, it is observed that string cosmological models corresponding to geometric string, Takabayasi string and Reddy string remain same. Equation (14, 15, 16) represents some physical and kinematical properties of the models. It can be easily seen that physical quantities like energy density, tension density of the string and the scalar field diverges as $T \to 0$. This behaviour is similar to that of the cosmic string model obtained by Reddy in four dimensional Lyra manifold.

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