Application Of Wavelet Transform For Fault Diagnosisof Rolling Element Bearings

P. G. Kulkarni, A. D. Sahasrabudhe

Abstract:- The rolling element bearingsare most critical components in a machine. Condition monitoring and fault diagnostics of these bearings are of great concern in industries as most rotating machine failures are often linked to bearing failures. This paper presents a methodology for fault diagnosis of rolling element bearings based on discrete wavelet transform (DWT) and wavelet packet transform (WPT). In order to obtain the useful information from raw data,db02 and db08 wavelets were adopted to decompose the vibration signal acquired from the bearing. Further De-noising technique based on wavelet analysis was applied. This de-noised signal was decomposed up to 7th level by wavelet packet transform (WPT) and 128 wavelet packet node energy coefficients were obtained and analyzed using db04 wavelet.The results show that wavelet packet node energy coefficients are sensitive to the faults in the bearing. The feasibility of the wavelet packet node energy coefficients for fault identification as an index representing the health condition of a bearing is established through this study.

Index Terms: - condition monitoring, de-noising, discrete wavelet transform, rolling element bearings, thresholding, vibration, wavelet packet transform.

————————————————————

1 INTRODUCTION

Prompt diagnostics of rolling element bearings fault is critical not only for the safe operation of machines, but also for the reduction of maintenance cost.The vibration based signal analysis is one of the most important methods used for condition monitoring and fault diagnostics of rolling element bearings because the vibration signal always carry the dynamic information of the system. The selection of proper signal processing technique is important for extracting the fault related information. Over the years with the rapid development in the signal processing techniques, for analyzing the stationary signals, techniques such as Fast Fourier Transform (FFT) and Short Time Fourier Transform (STFT) are well established. Fourier analysis is one of the classical tools to convert data into a form that is useful for analyzing frequencies. The Fourier coefficients of the transformed function represent the contribution of each sine and cosine function at each frequency. Tandon and Choudhury [1] presented a detailed review of vibration and acoustic measurement methods for detection of defects in rolling element bearings. They have considered both localized and distributed defects. Pitting, spalling etc. are the examples of localized defects while waviness, surface roughness, misaligned races are the examples of distributed defects. Detailed description of these defects is available in standard books on bearings [2, 3]. McFadden and Smith [4, 5] have developed a model to describe the vibrations produced by a single pointdefect and multi point defects on the inner race of a rolling element bearing under constant radial load.

- *P. G. Kulkarni is currently pursuing Ph. D. degree program in Mechanical engineering in University of Pune, India, PH-+919890265462.*
  E-mail: prof.pgk@mail.com
- *A. D. Sahasrabudhe is Director, College of Engineering, Pune, India, PH-+919423582025.*
  E-mail: director@coep.ac.in

It was concluded that frequency components related to the element passing frequency were not the largest components in each group in the spectrum of multi point defects.In addition to local and distributed defects causing vibrationin bearings, variation in stiffness of bearings give rise to vibration. Different causes of bearings vibration are discussed in [6].Sunnersjo [7] has carried out study on the effect of varying compliance on vibrations of rolling bearings with emphasis on radial vibrations with positive clearance. In addition to FFT spectrum analysis, various researchers have used time domain methods for vibration monitoring of rolling bearings.Tandon [8] has compared vibration parameters such as overall RMS, peak, crest factor, cepstrum etc. for the detection of defects in rolling element bearings. Heng and Nor [9] carried out the statistical analysis of sound and vibration signals for monitoring the condition of rolling element bearings. The main drawback of the statistical analysis for rolling bearings is inability to identify the location of faults.Su and Lin [10] extended the vibration model developed by McFadden and Smith to describe the bearing vibration under diverse loading. They have reported the need of time domain analysis alongwith frequency domain to reliably monitor a running bearing.McFadden and Smith [11] explained the step by step procedure of applying high frequency resonance technique (HFRT) for bearing defect detection. This study shows that conventional spectrum analysis cannot detect bearing defects in the presence of vibration from gear and other machine elements. JayaswalPratesh et al.[12] investigated the feasibility of FFT and band-pass analysis for fault detection in rolling element bearings with multiple defects. Fourier transform approach works fine for the analysis of signals that are produced by some periodic process. Most of the signals encountered in practice are finite and aperiodic. The discrete Fourier transform is difficult to adapt to such practical situations. Secondly, this technique has limited success when the signal is buried in background noise or when the signal-to-noise ratio is small. Wavelet Transform (WT) has been viewed as an attempt to overcome shortcomingsof Fourier transform. The basic idea is to choose a basis function having zero mean called “mother wavelet”. Peng and Chu [13] presented a detailed review on the application of wavelettransform in machine fault diagnostics. Wavelet transform while performingtime-frequency analysis is best suited to extract fault features, de-noising and extraction of weak signals and singularity detection. Lin et al.[14] have proposed method for wavelet threshold de-noising employing Morlet wavelet. Wang et al. [15] have presented multiwavelet de-noising method with improved neighboring coefficients. They have reported that de-noising method with improved neighboring coefficients has better noise cancellation ability than other methods.Chen and Gao [16] have studied de-noising and feature extraction techniques based on wavelet analysis. They have proposed improved wavelet thresholding algorithm to eliminate noise from vibration signals and applied this algorithm for structure health monitoring system. Junsheng et al. [17] proposed scale-wavelet power spectrum comparison and auto-correlation analysis of time-wavelet power spectrum for constructing impulse response wavelet.Chebil et al.[18] presented a wavelet based analysis technique using discrete wavelet transform and the discrete wavelet packet transform for the diagnosis of faults in rotating machinery. Rafiee et al. [19] discussed selection of mother wavelet for gear and bearing fault diagnosis along with automatic feature extraction system. Prabhakar et al. [20] have investigated the diagnosis of single and multiple ball bearing race faults by discrete wavelet transform. The wavelet transform (WT) decomposes a signal into a representation comprised of local basis functions called wavelets. Each wavelet is situated at a different position on the time axis. Any particular local feature of a signal can be identified from the scale and position of the wavelets decomposed. Advantages of wavelet analysis lie in its ability to examine local data with a "zoom lens having an adjustable focus" to offer multiple levels of details and approximations of the original signal. This paper presents a methodology for fault diagnosis of rolling element bearings based on discrete wavelet transform (DWT) and wavelet packet transform (WPT).

The paper is organized as follows:
In section 2, the concept of wavelet transform, wavelet packet transform along with wavelet based de-noising is reviewed. Section 3 discusses the data acquisition and experimental set up for obtaining the bearing vibration data and the instrumentation for data acquisition. Section 4presents the results of wavelet transform, wavelet packet transform and wavelet based de-noising. The last section concludes the paper.

2 WAVELET TRANSFORM

Wavelet theory has emerged as asignal processing tool in many fields and has many distinct merits. It was first put forward by Morlet in 1984. Wavelets are mathematical functions that cut up data into different frequency components but different from short time Fourier transform (STFT) in that each component is studied with a resolution matched to its scale. They are suitable for analyzing physical situations where the signal contains discontinuities and sharp spikes. The commonly used wavelet algorithms are continuous wavelet transform (CWT), discrete wavelet transform (DWT) and wavelet packet transform (WPT).

2.1Continuous Wavelet Transform

The continuous wavelet transform is dot product of x (t) with translate and dilate of a wavelet ψ. ψ is wavelet translated by b and dilated by a.

$$CWT(b,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)dt \qquad (1)$$

Where ψ*(t) stands for the complex conjugation of ψ (t).

Above is CWT of function x€ L2(R) w.r.t. wavelet ψ evaluated at translation b and dilation a. Equation (1) indicates that the wavelet analysis is a time-frequency analysis, or a time-scaled analysis. The analyzing function or windowing function ψ must satisfy certain admissibility conditions to be considered for wavelet analysis.The dilation parameter a, translation parameter b are also referred as the scaling and shifting parameters. By changing the value of dilation parameter a, the portion of the function in vicinity of t=b can be examined in different resolutions (referred as multi-resolution analysis). By changing the value of translation parameter b, the function around the point t=bcan be examined by the wavelet window piece by piece.It is possible to reconstruct the original function from its wavelet transform. The inversion formula [21] is given by:

$$x(t) = \frac{1}{c_\psi} \iint w(a,b)\psi_{(a,b)}(t)\frac{dadb}{a^2} \qquad (2)$$

where $c_\psi = \int_{-\infty}^{+\infty} \frac{|\psi(\omega)|^2}{|\omega|} d\omega < \infty$

Using above equation, the original signal can be reconstructed without any loss of data. Scaling parameter a is positive real and translation parameter b is positive or negative.At high frequencies, the wavelet reaches at a high time resolution but a low frequency resolution, whereas, at low frequencies, high-frequency resolution and low time resolution can be obtained.

2.2 Discrete Wavelet Transform

The CWT is defined at all points in the plane and corresponds to a redundant (extra) representation of the information present in the function. This redundancy requires a large amount of computation time. Instead of continuously varying the parameters, we analyze the signal with a small number of scales with varying number of translations at each scale. The discrete wavelet transform may be viewed as a "discretization" of the CWT through sampling specific wavelet coefficients. A critical sampling of the CWT given by equation (1) is obtained via $a=2^{-j}$ and $b=k2^{-j}$, where j and k are integers representing the set of discrete dilations and translations respectively. Upon this substitution, discrete wavelet transform is obtained and is given by:

$$W(j,k) = \int_{-\infty}^{+\infty} x(t)2^{j/2}\psi(2^j t - k)dt \qquad (3)$$

The term critical sampling denotes the minimum number of coefficients sampled from CWT to ensure that all the information present in the original function is retained by the wavelet coefficients [22].The DWT computes the wavelet coefficients at discrete intervals (integer power of two) of time and scales. In discrete wavelet transform, the signal is decomposed into a tree structure of low and high pass filters.

Each step transforms the low pass filter into further lower and higher frequency components as shown in Fig. 1.

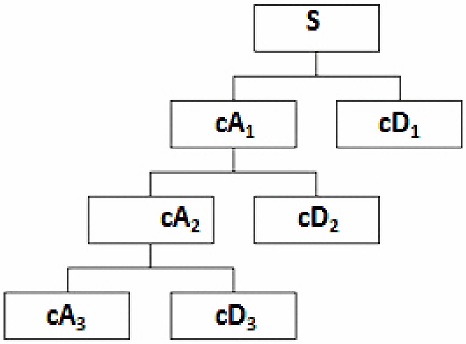


Fig.1. DWT decomposition tree of three-level [23]

The frequency band of each filter depends on the decomposition level. The high frequency components are not analyzed further. The low pass filter produces approximation coefficients and high pass filter produces detail coefficients. For example, if N_t=Total length of signal, j=DWT decomposition level, Fs=Sampling frequency, then each vector contains $Nt/2^j$coefficients. Approximation corresponds to Frequency band $[0, Fs/2^{j+1}]$ while detail covers the frequency range $[Fs/2^{j+1}, Fs/2^j]$.

At any decomposition level, the signal can be expressed as the sum of approximation and detail coefficients as follows:

$$S = A_j + \sum D_i (i \leq j) \quad (4)$$

whereAj=Approximation coefficients at jth level
Di=Detail coefficients

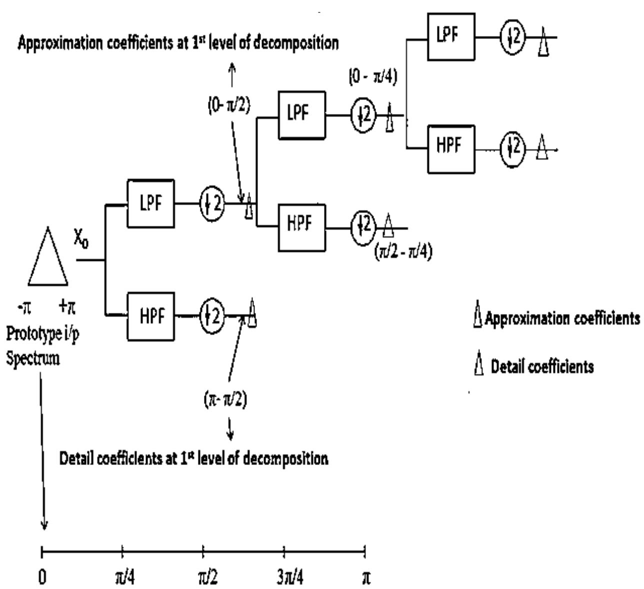


Fig.2. Filter bank representation of DWT

Fig. 2 shows the prototype spectrum of frequency range (0-π) in which the frequency band of low and high pass filter at each decomposition level is shown.

2.3 Wavelet Packet Transform

Wavelet packet transform (WPT) decomposes not only the approximation coefficients but also the detail coefficients.In Fig. 3, an example of a wavelet packet decomposition tree of three levels is illustrated. The sampling rate of the signal is 24 kHz. The frequency sub-band at each node of the wavelet packet tree is shown.

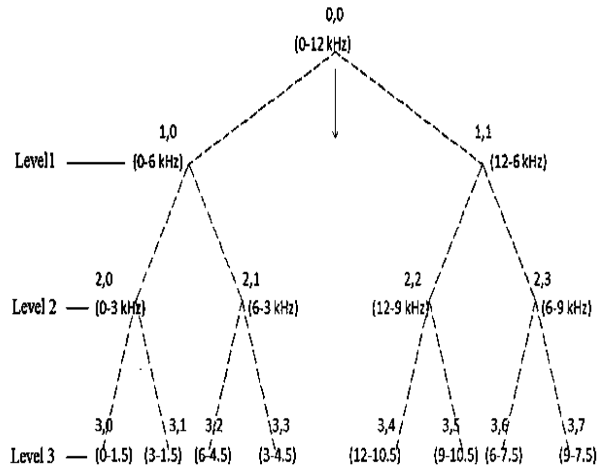


Fig.3. Wavelet packet tree

A split on detail coefficients leads to change in basis set and these basis sets are called wavelet packets. Wavelet packets are a collection of functions given by [24]:

$$\left\{ 2^{-j/2} W_n (2^{-j} t - k), n \in N, j, k \in Z \right\} \quad (5)$$

Above function is generated from the following sequence of functions:

$$W_{2n}(t) = \sqrt{2} \sum_i h_1 W_n (2t - l) \quad (6)$$

$$W_{2n+1}(t) = \sqrt{2} \sum_i g_1 W_n (2t - l) \quad (7)$$

where h and g are the quadrature mirror filters, $W_0(t)$ and $W_1(t)$ are the scaling function and basic wavelet, respectively. The wavelet packet $\{2^{-j/2}$ Wn $(2^{-j} \mathbf{t} - \mathbf{k})\}$is a localized function of unit energy with scale 2^j, translation $2^j k$, and an oscillation parameter of n.

Time scale domain signal energy shows the similarity between signal and wavelet which is selected. Total energy can be obtained by:

$$E(n) = \sum_{i=1}^{n} |x[n]|^2 \quad (8)$$

where n=No. of samples of the signal

An appropriate wavelet selected as the base wavelet, must have maximum amount of energy of the wavelet coefficients.

2.4 Wavelet-Based Signal De-Noising

Wavelet threshold de-noising has been widely used and it was first proposed by Donoho[25].Wavelet decomposition of a signal is analogous to use filters that act as averaging filters producing approximations and others that produce details. If these details are small, they may be omitted without affecting main features of the signal. The underlying model for the noisy signal is of the form:

$$S[n] = x[n] + N_1[n] \qquad (9)$$

The objective of wavelet de-noising is to supress the additive noise $N_1[n]$ from a signal S[n] in three steps:

1. Signal decomposition: Signal S[n] isdecomposedinto j level of wavelet transform and coefficients are calculated.
2. Thresholding: Then the threshold is selected and the detail parts through wavelet transform are compared with the threshold and the detail parts are set to zero if they are less than the threshold.
3. Signal reconstruction:Finally the signal is reconstructed using the original approximation coefficients of level j and modified detail coefficients.

Generally there are two kinds of threshold functions viz., hard thresholding function and soft thresholding function. These are examples of shrinkage rules. The form of universal hard threshold function is:

$$S=\begin{cases} \mathbf{x} & |\mathbf{x}| \geq \mathbf{T} \\ \mathbf{0} & |\mathbf{x}| < \boldsymbol{T} \end{cases} \qquad (10)$$

In hard thresholding, the coefficients whose absolute values are lower than the threshold are set to zero.

The form of universal soft threshold function is:

$$S=\begin{cases} \mathbf{x}-\mathbf{T} & \mathbf{x} > T \\ \mathbf{0} & |\mathbf{x}| < \boldsymbol{T} \\ \mathbf{x}+\mathbf{T} & \mathbf{x} < -T \end{cases} \qquad (11)$$

Soft thresholding is an extension of hard thresholding by first setting to zero the coefficients whose absolute values are lower than the threshold and then shrinking the nonzero coefficients towards zero.

3 DATA ACQUISITION

In the present work, the vibration signatures were collected from the bearing of an experimental set up as shown in Fig.4. The shaft of the experimental setup is driven by an AC motor through a gear coupling. The test bearing SKF 6205 was mounted in the bearing casing on the shaft and loaded by screw and nut arrangement in radial and axial direction.The vibrations of the bearing were recorded using PCB tri-axial shear accelerometer with NI 9234 sound and vibration module. Vibration signals were acquired at different speeds up to 3000 rpm of the system for both defect free and defective bearing. The defects were created on inner race, outer race and rolling elements by electric discharge machining. The parameters of SKF 6205 bearing are: Number of balls 9, diameter of balls 8.5mm, pitch diameter 38.5mm and contact angle 0^0.

Fig.4 Experimental setup

The vibration signal was acquired for the analysis of three test bearings at different speeds. The details about the test bearings and size of defects are shown in Table 1.

TABLE 1
DEFECT DETAILS FOR TEST BEARINGS

Sr. No.	Bearing Defect	Defect Size	Remark
1	No defect	------	------
2	Inner Race	1 mm	One Defect
3	Ball+Outer Race	1mm each	Two defects

The theoretical characteristic frequencies for above cases of defect are calculated and these frequencies are shown in Table 2 at different speeds.

TABLE 2
THEORETICAL CHARACTERISTIC FREQUENCIES(HZ)

Speed in rpm	fs	BPFO fod	BPFI fid	BPFR fbd
2100	35	122.5	192.2	168
2400	40	139.2	220	192
2700	45	156.6	247	216
3000	50	175	274.5	240

4 WAVELET BASED FEATURE EXTRACTION

4.1 Multi-resolution Analysis

The vibration signal from defect free bearing running at 3000 rpm is acquired in the initial stage shown in Fig. 5.

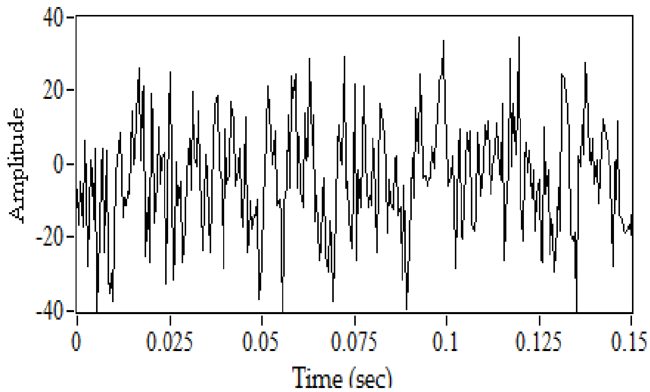


Fig.5 Time domain waveform of defect free bearing vibration signal at 3000 rpm

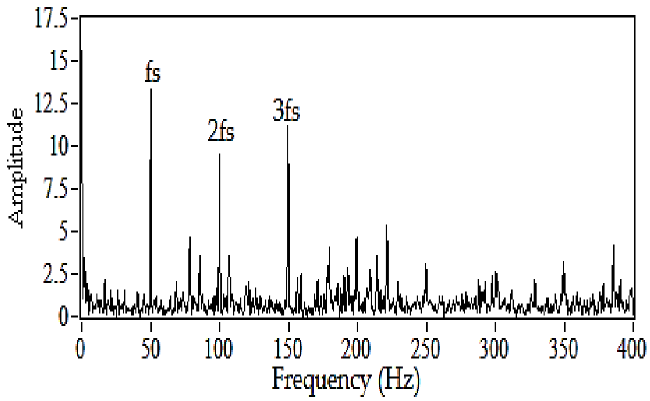


Fig.6 Spectrum defect free bearing vibration signal at 3000 rpm (fs: shaft rotation frequency and its harmonics)

The spectrum of the vibration signal of the defect free bearing shows the peak at shaft rotational frequency and its harmonics as shown in Fig.6.The following sections 4.1.1 and 4.1.2 presents the results of multi-resolution analysis for different defect conditions of the bearing.

4.1.1 Defect on outer race and ball

Time domain waveform of a defective bearing with defect on outer race and ball is acquired at 3000 rpm and is shown in Fig. 7.

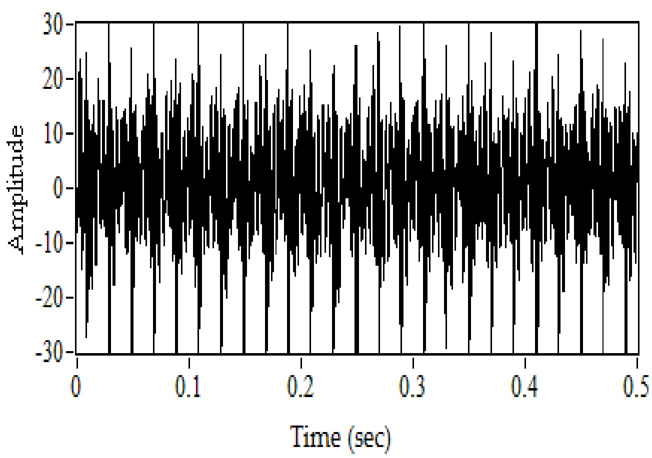
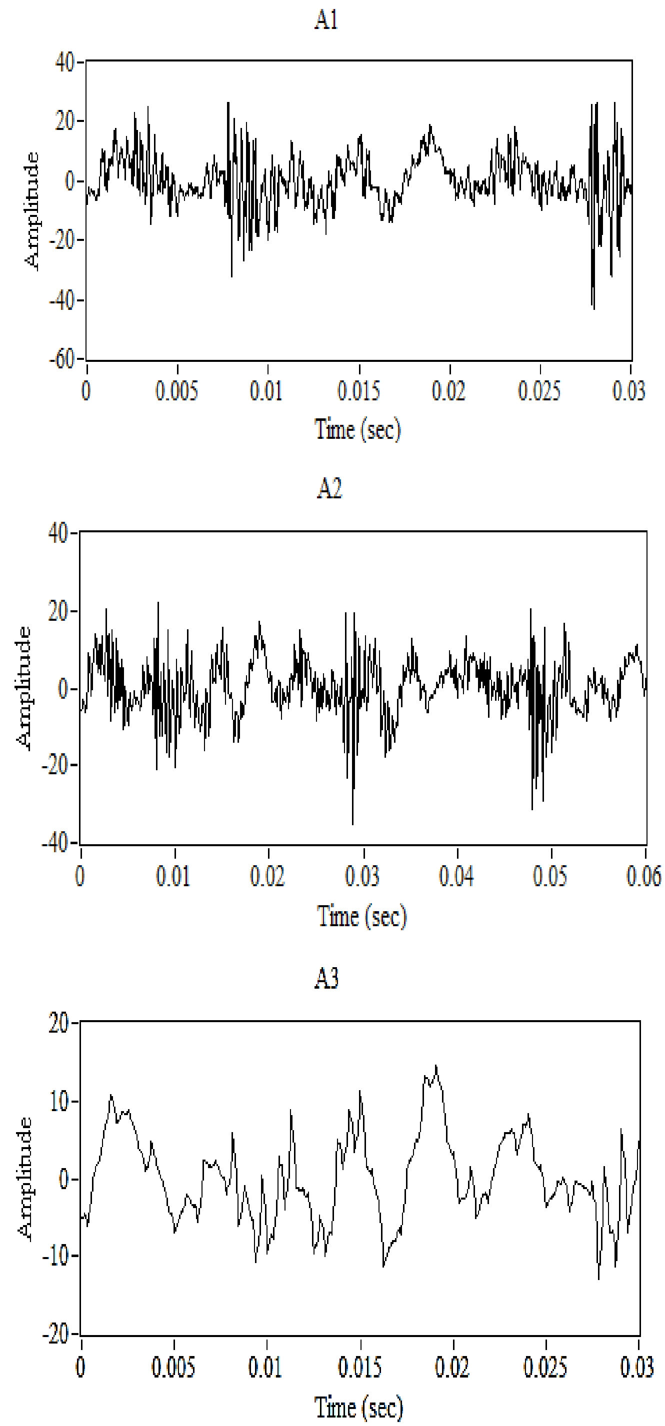


Fig.7. Time domain waveform of bearing vibrationsignal with defect on outer race and ball at 3000 rpm

Above waveform does not indicate the presence of fault. Hence this signal is decomposed up to 5th level by using db02 mother wavelet. The result of this analysis is shown in Fig.8 and Fig.9 A1 to A5 in Fig. 8 means approximations of the signal from 1st to 5th level and D1 to D5 in Fig. 9 means details of the signal from 1st to 5th level.



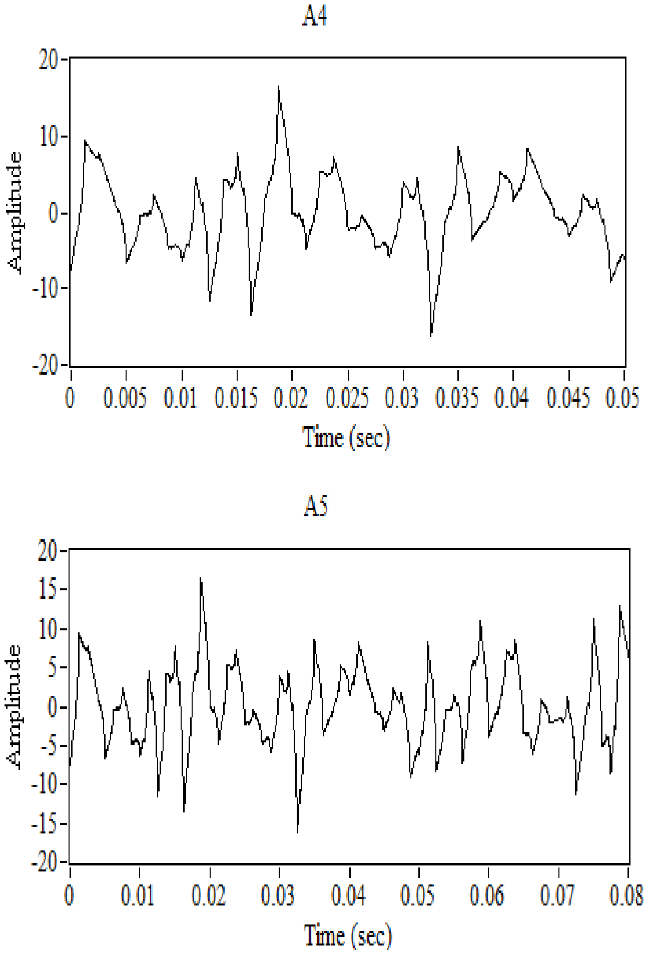
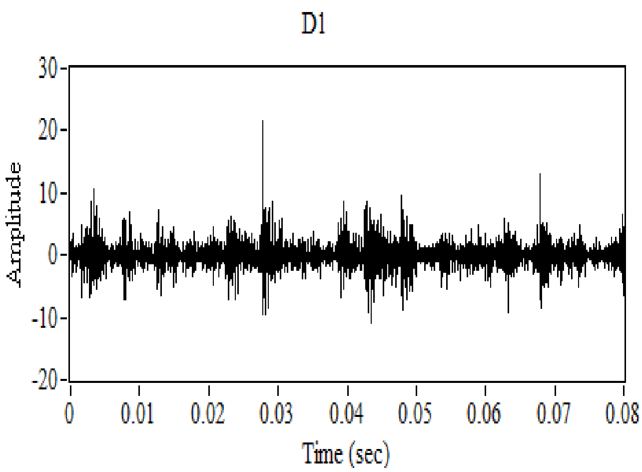
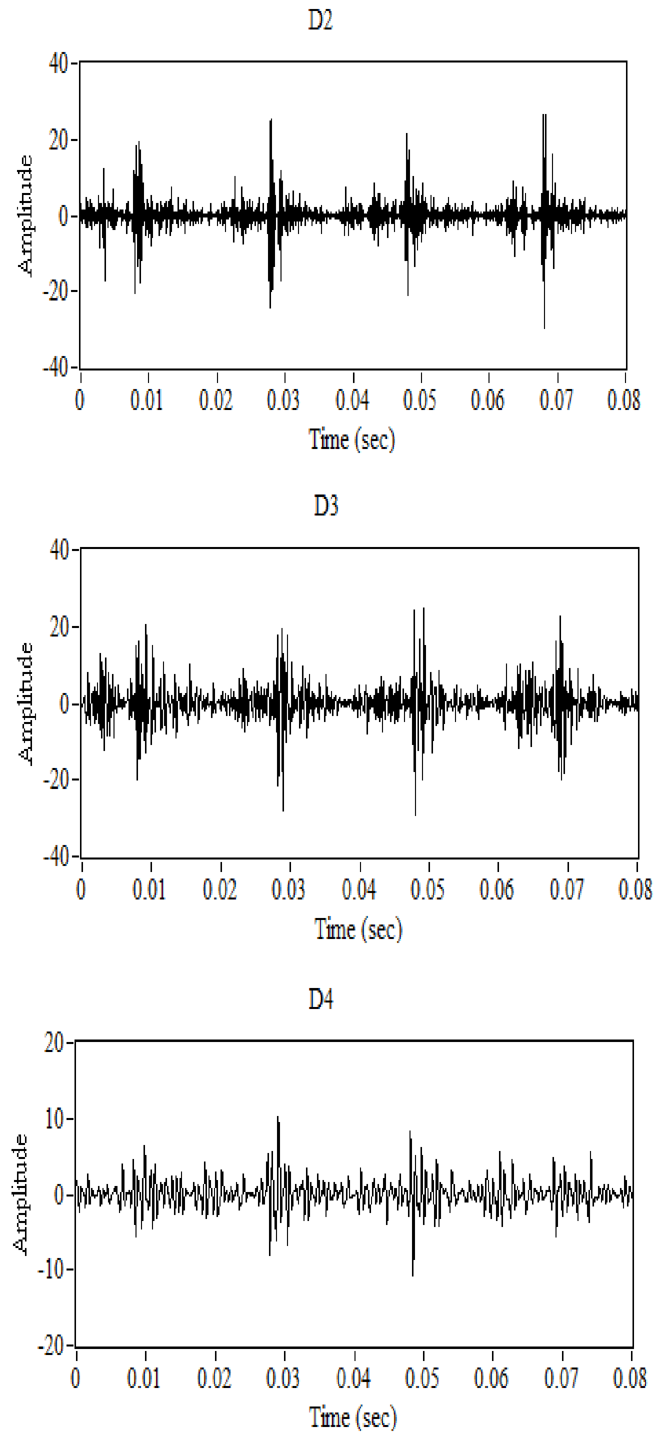


Fig.8 Approximations of the signal from A1 to A5using db02 wavelet





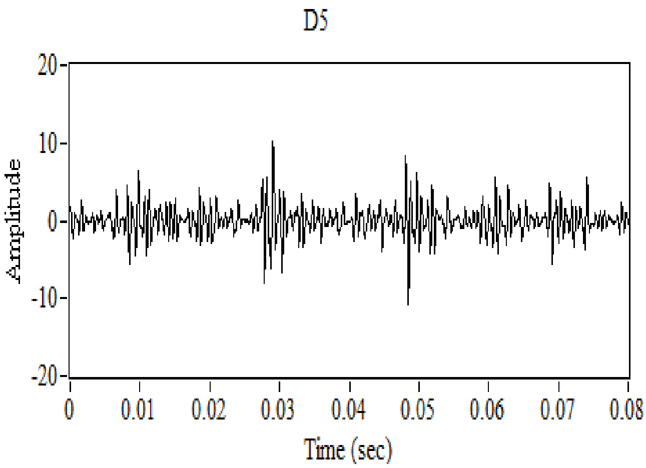


Fig.9 Details of the signal from D1 to D5using db02 wavelet

The approximation of the signal at the 5th level (A5) contains the information related to faults in the bearing. Hence the approximation signal at this level is analysed by spectrum analysis. The result of FFT is shown in Fig. 10.

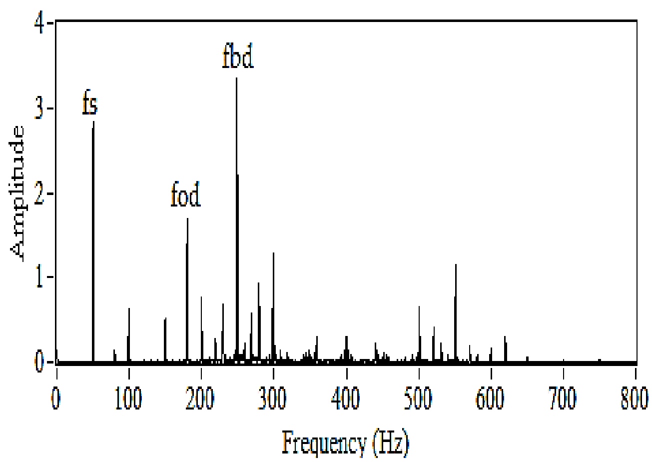


Fig.10Spectrum of the approximation of the signal at5th level using db02(fs: shaft rotation frequency, fod: outer race defect frequency, fbd: ball defect frequency)

The theoretical fault frequencies related to defect at the outer race and ball from Table 2 are 175 Hz and 240 Hz at 3000rpm respectively. It can be found that the spectrum in Fig. 10 shows the presence of faults on the outer race and ball on the bearing. The magnitude of approximation and detail coefficients depends on how closely the analyzing or tool function (mother wavelet) matches with the signal to be analyzed. To investigate the effect of using higher order wavelet on the results of multi-resolution analysis, db08 mother wavelet was used and above analysis was carried out. The approximation at the fifth level and the corresponding spectrum is shown in Fig. 11 and Fig. 12 respectively. It is observed that theanalyzing function db08 hasbetter

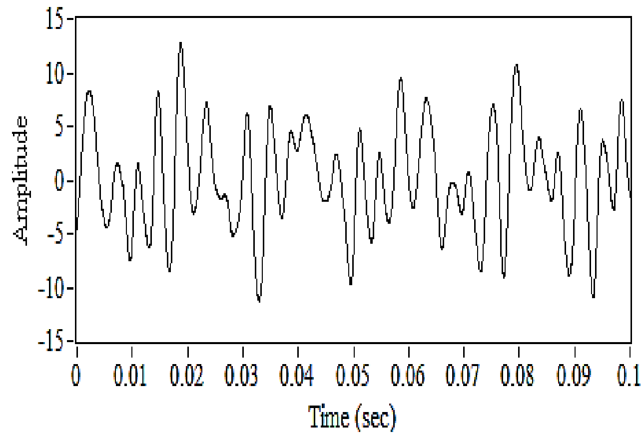


Fig. 11 Approximation of the signal at 5th level using db08

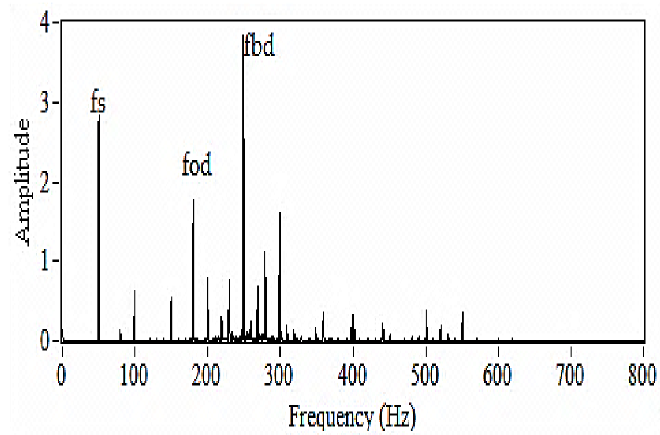


Fig. 12 The spectrum of the approximation of the signal at 5th level using db08 (fs: shaft rotation frequency, fod: outer race defect frequency, fbd: ball defect frequency)

matching with the vibration signal. This is due to the fact that higher order mother wavelet db08 has more no. of vanishing moments than db02.

4.1.2 Defect on inner race

Vibration signal from defective bearing with defect on inner race is acquired at 3000 rpm and decomposed up to level 5 by using db02 and db08 mother wavelet. Fig. 13 shows this time waveform signal.

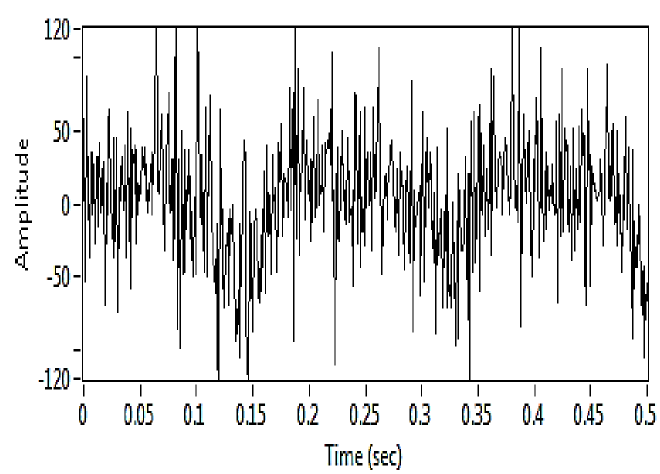


Fig.13 Time domain waveform of bearing vibration signal with defect on inner race at 3000 rpm

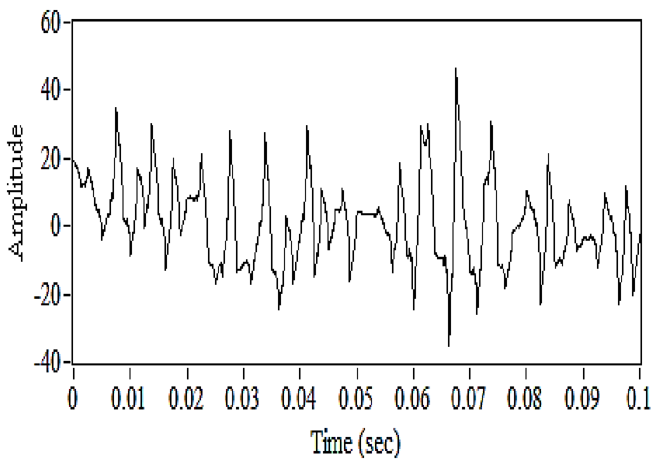


Fig. 14 Approximation of the signal at 5th level using db02

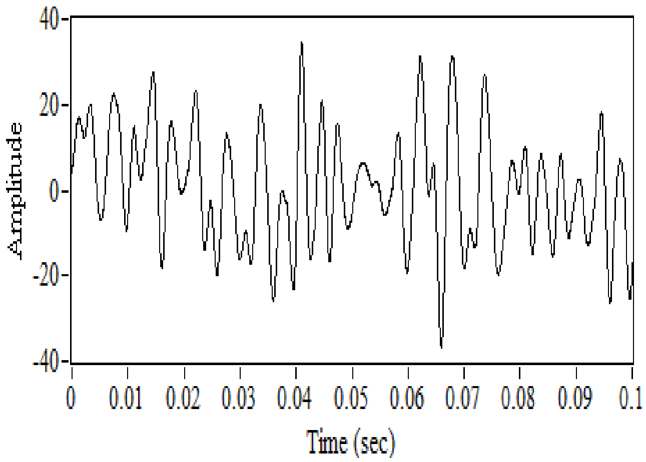


Fig. 15 Approximation of the signal at 5th level using db08

Similar to previous case of defect discussed in section 4.1.1, as seen from Fig. 15, db08 wavelet shows better agreement with the vibration signal acquired from bearing with inner race defect. The spectrum of this vibration signal shows the presence of inner race defect as shown in Fig. 16 with 4% deviation from the theoretical defect frequency of 274.5 Hz.

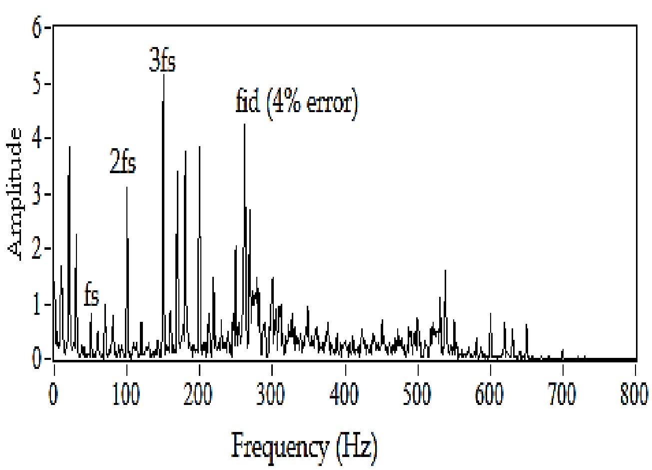


Fig.16 Spectrum of the approximation of the signal at 5th level using db02 (fs: shaft rotation frequency and its harmonics, fid: inner race defect frequency)

4.2 Wavelet based de-noising

Performance of thresholding on de-noising a signal is evaluated first on the simulated signal and then on the vibration signal acquired from a bearing in the noisy environment.

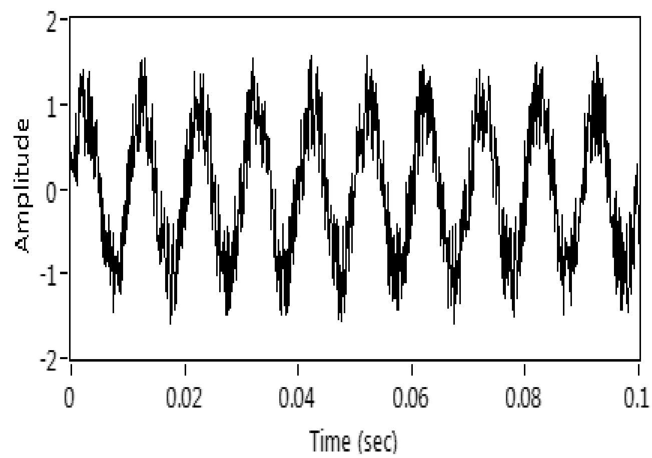


Fig. 17a) Signal containing reference signal and noise.

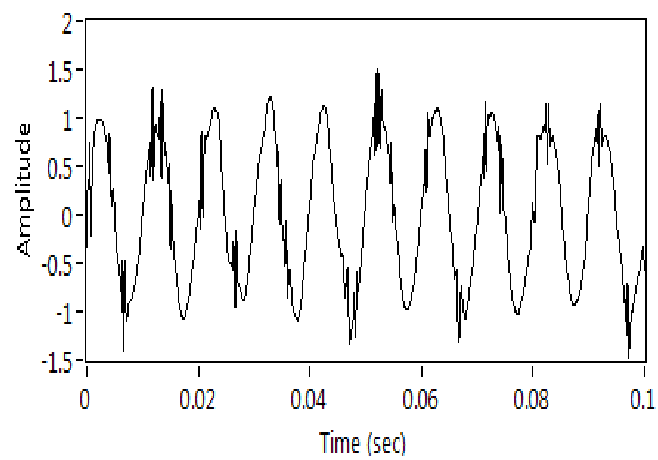


Fig. 17b) The de-noised results of hard thresholding.

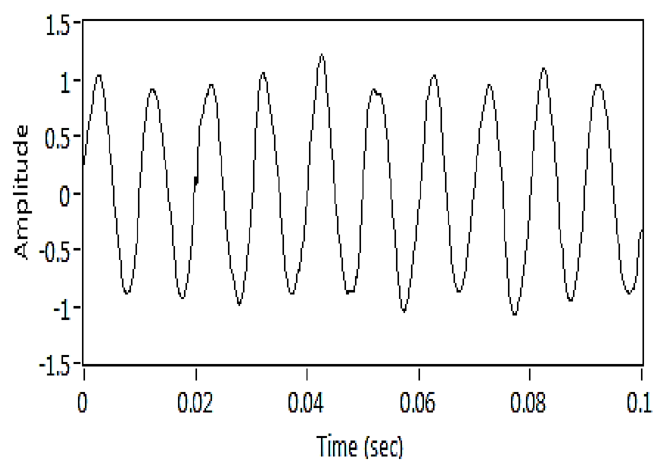


Fig. 17c) The de-noised results of soft thresholding.

Fig. 17(a) shows the simulated signal with noise. The de-noising results of hard and soft thresholding are shown in Fig. 17(b) and Fig. 17(c) respectively. Both methods have recovered the signal. In case of soft thresholding, high frequency part of the signal is filtered resulting in the loss of the useful information. The signal reconstructed by hard threshold shows some noisy areas. To introduce noise while acquiring the vibration signal from bearing, the worn out sleeve of a gear coupling shown in Fig.18 connecting the motor shaft and the bearing shaft is used. This worn out sleeve results in play with the meshing gears mounted on the shafts resulting in increase in the overall vibration level and also produces high noise level.

Fig.18 Worn out coupling sleeve

Fig. 19(a) and (b)shows the time domain signal in the noisy environment acquired from a bearing with outer race defect at 3000 rpm and its FFT spectrum respectively.

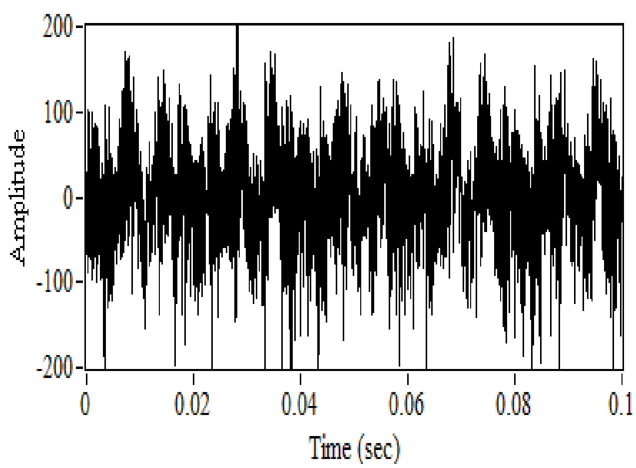


Fig. 19a) Time Domain signal from bearing with outer racedefect at 3000 rpm.

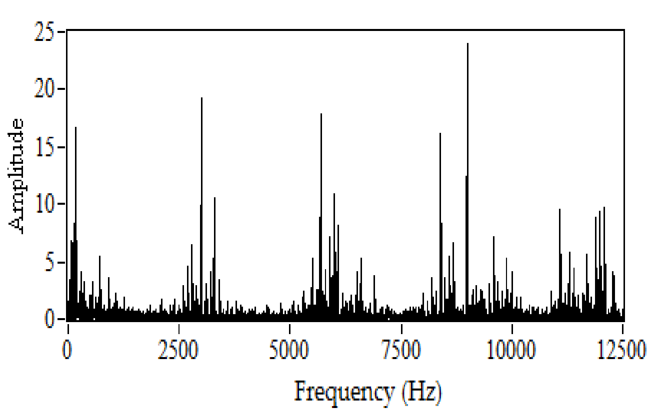


Fig. 19b) Spectrum of signal before de-noising.

De-noising of above signal is carried out by hard and soft thresholding. The time domain waveform after de-noising is shown in Fig. 20.The results are shown in Fig. 21(a) and (b) respectively.Similar to the simulated signal, soft thresholding results in the loss of the useful information and hard thresholding showing some noisy areas.

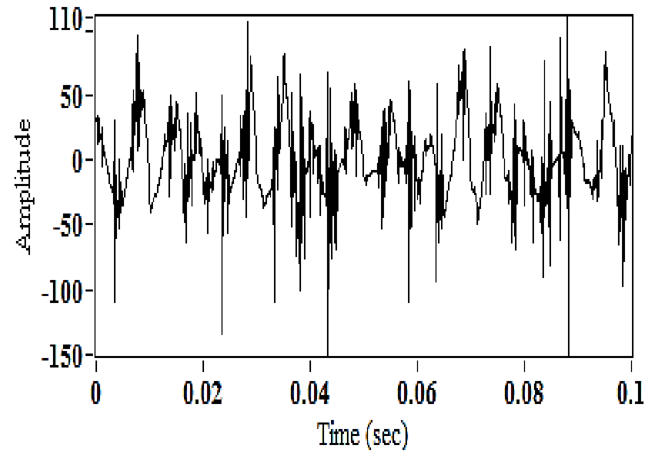


Fig.20 Time Domain signal from bearing with outer race defect at 3000 rpm after de-noising.

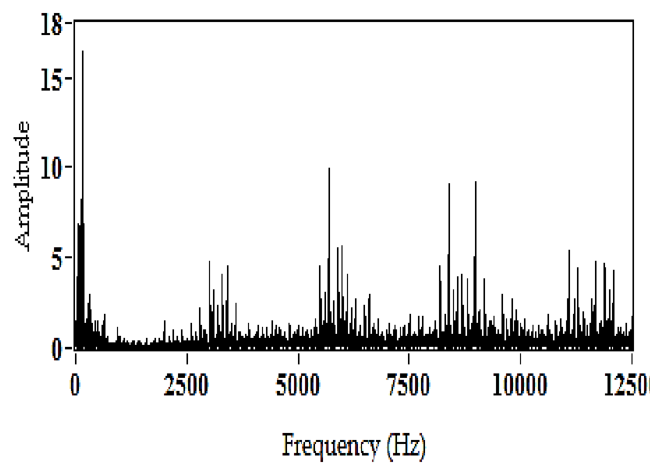


Fig. 21a) Spectrum after hard thresholding.

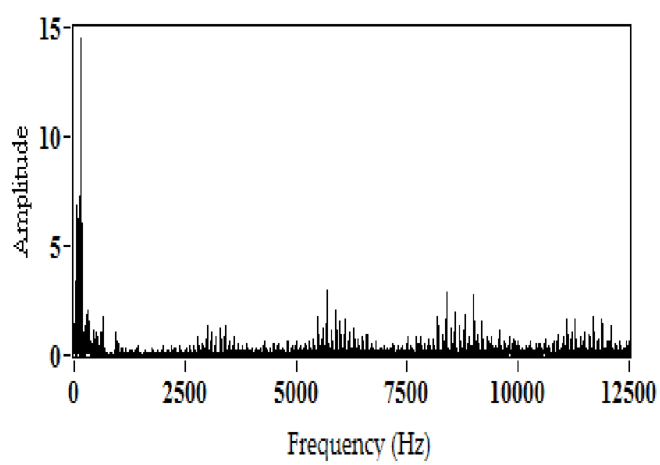


Fig. 21b) Spectrum after soft thresholding.

4.3 Wavelet packet node energy coefficients

Further the effect of de-noising the vibration signal acquired from bearing with defect on outer race on wavelet packet node energy coefficients was investigated using db04 wavelet. To start with, wavelet packet node energy coefficients of the signal without de-noising were found as shown in Fig. 22. No. of nodesare2^j i.e.128 where j is the no. of decomposition levels which is 7 in this case. Each level corresponds to a specific band of frequency as discussed in section2.2.

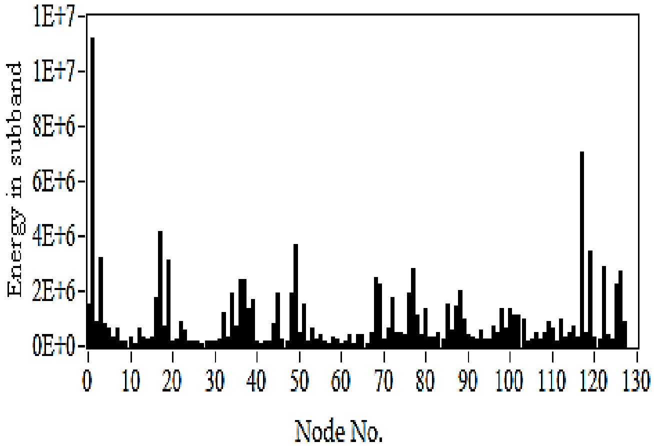


Fig. 22 Wavelet packet node energy coefficients ofvibration signal before de-noising.

Afterde-noising the signal, energy in the node coefficients was found to reduce as shown in Fig. 23.

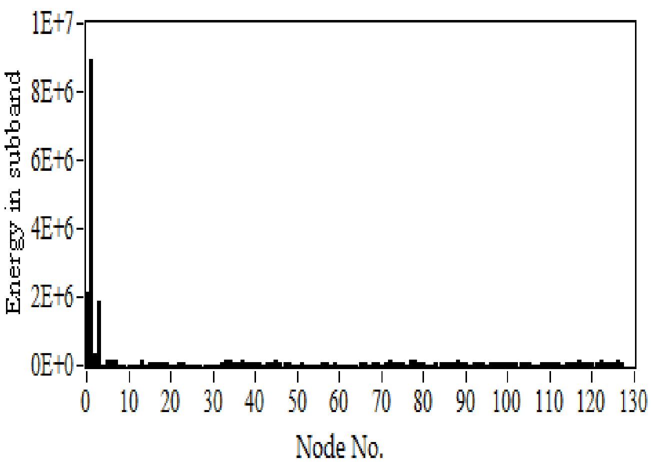


Fig.23 Wavelet packet node energy coefficients afterde-noising

It is clear that the de-noising results in reduction in the energy of a signal which corresponds to noise and in a way the portion of the signal carrying important fault related information is retained. Fig. 24 shows energy at decomposition levels from 1 to 6.

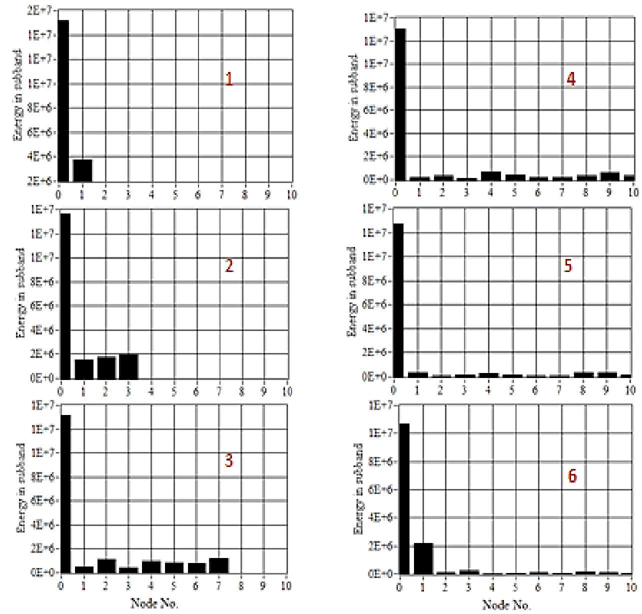


Fig.24 Wavelet packet node energy coefficients from1st to 6th level.

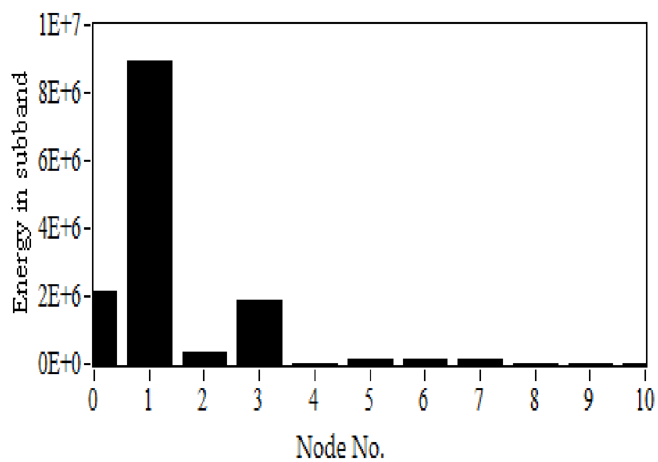


Fig. 25 Wavelet packet node energy coefficients at 7^{th}level.

Node no. 1at 7th decomposition level(frequency band 187.5-93.7 Hz) corresponds to the outer racefault frequency of 175 Hz. Energy at this node should show a high value. The same is observed as shown in Fig. 25.Above analysis shows the sensitivity of wavelet packet node energy coefficients to the faults in the bearing. Hence the wavelet packet node energy coefficients areuseful forfault identification as an indexrepresenting the health condition of a bearing.Multiresolution analysis, de-noising and WPT analysis is carried out for all the cases of bearing shown in Table 1 at different speeds. However the results for bearing vibration signal acquired at 3000 rpm are presented in this paper.

5 CONCLUSION

Wavelet transform based time-frequency analysishas great advantages in dealing with the vibration data of the bearing health monitoring system. Wavelet transform can be effectively used for de-noising the bearing vibration signal corrupted due to noisy environment. The success of de-noising depends on appropriate decomposition criteria and selection of suitable wavelet threshold. Further wavelet transform decomposes the non-stationary bearing vibration signal into components with simple frequency content. The wavelet packet node energy coefficients are sensitive to the faults in the bearing. The feasibility of using the wavelet packet node energy coefficients for fault identification as an index representing the health condition of a bearing is established through this study.

ACKNOWLEDGMENT
The authors wish to thank Board of College and University Department (BCUD) Pune. This work was supported in part by a grant from BCUD Pune.

REFERENCES
[1]. N. Tandon,A. Choudhury,"A review of vibration andacoustic measurement methods for the detection of defectsin rolling element bearings,"*Tribology International*, vol. 32, pp. 469-480, 1999.

[2]. T. S. Nisbet, G. W. Mullett, *Rolling Bearings in Service: Interpretation of Types of Damage.*Hutchinson, 1978.

[3]. T. A. Harris, *Rolling Bearing Analysis.* John Wiley and Sons, New York, 2001.

[4]. P. D. McFadden and J. D. Smith, "Model for thevibration produced by a single point defect in a rollingelement bearing,"*Journal of Sound and Vibration,*vol. 96, no.1, pp. 69-82,1984.

[5]. P. D. McFadden and J. D. Smith, "The vibration produced by multiple point defects in a rolling element bearing," *Journal of Sound and Vibration,* vol. 98, no.2, pp. 263-273, 1985.

[6]. Alexej Barkov and Natalja Barkova, "Condition assessment and life prediction of rolling element bearings-part I," *Vibroacoustical Systems and Technologies,* St. Petersburg, Russia, 1995.

[7]. C. S. Sunnersjo,"Varying compliance vibrations of rolling bearings,"*Journal of Sound and Vibration*, vol. 58, no.3, pp. 363-373, 1978.

[8]. N.Tandon, "A comparison of some vibration parameters for the condition monitoring of rolling element bearings," *Measurement*, vol. 12, pp. 285-289, 1994.

[9]. R. B. W. Heng and M. J. M. Nor,"Statistical analysis of sound and vibration signals for monitoring rolling element bearing condition,"*Applied Acoustics*, vol. 53, no.1-3, pp. 211-296, 1998.

[10]. Y.T. Su and S. J. Lin, "On initial fault detection of a tapered roller bearing: frequency domain analysis,"*Journal of Sound and Vibration*, vol. 155, no.1, pp. 75-84, 1992.

[11]. P.D. McFadden and J. D. Smith, "Vibration monitoring of rolling element bearings by the high frequency resonance technique-a review,"*Tribology International*, vol. 17, no.1, pp. 3-10, 1984.

[12]. PrateshJayaswal, A. K. Wadhwani and K. B. Mulchandani, "Rolling element bearing fault detection via vibration signal analysis,"*The Icfai University Journal of Science &Technology,*vol. 4, no.2, pp. 7-20, 2008.

[13]. Z. K. Peng, F. L. Chu, "Application of the wavelet transform in machine condition monitoring and fault diagnostics: a review with bibliography,"*Mechanical Systems and Signal Processing,*vol. 18, pp. 199-221, 2004.

[14]. Jing Lin, Ming J. Zuo, Ken R. Fyfe, "Mechanical fault detection based on the wavelet de-noising technique,"*Journal of Vibration and Acoustics*, vol. 126, pp.9-16, 2004.

[15]. Xiaodong Wang, Yanyang Zi, Zhengjia He, "Multiwavelet denoising with improved neighboring coefficients for application on rolling bearing fault diagnosis,"*Mechanical Systems and Signal Processing*, vol. 25, pp. 285-304, 2011.

[16]. Xiang-jun Chen, Zhan-feng Gao, "Data processing based on wavelet analysis in structure health monitoring system,"*Journal of Computers,*vol. 6 no.12, pp. 2686-2691, 2011.

[17]. Cheng Junsheng, Yu Dejie, Yang Yu, "Application of an impulse response wavelet to fault diagnosis of rolling bearings," *Mechanical Systems and Signal Processing,*vol. 21, pp. 920-929, 2007.

[18]. J. Chebil, G. Noel, M. Mesbah, M. Deriche, "Wavelet decomposition for the detection and diagnosis of faults in rolling element bearings,"*Jordan Journal of Mechanical and Industrial Engineering*, vol.3, no.4, pp. 260-267 2009.

[19]. J. Rafiee, M. A. Rafiee, P. W. Tse, "Application of mother wavelet functions for automatic gear and bearing fault diagnosis,"*Expert Systems with Applications,*vol. 37, pp. 4568-4579, 2010.

[20]. S. Prabhakar, A.R. Mohanty, A.S Sekhar, "Application of discrete wavelet transform for detection of ball bearing race faults,"*Tribology International,*vol. 35, pp. 793–800, 2002.

[21]. HaiQiu, Jay Lee, Jing Lin, Gang Yu, "Wavelet filter-based weak signature detection method and its application on rollingelement bearing prognostics,"*Journal of Sound and Vibration*, vol. 289, pp. 1066-1090, 2006.

[22]. K. P. Soman, K. I. Ramachandran, *Insight into Wavelets-From Theory to Practice*. Prentice Hall of India Private Limited, 2005.

[23]. N. G. Nikolaou, I. A. Antoniadis, "Rolling element bearing fault diagnosis using wavelet packets,"*NDT&E International*, vol. 35, pp. 197-205, 2002.

[24]. B.Liu, "Selection of wavelet packet basis for rotating machinery fault diagnosis,"*Journal of Sound and Vibration,*vol.284, pp. 567-582, 2005.

[25]. D. L. Donoho, "De-noising by soft-thresholding,"*IEEE Transactions onInformation Theory,* vol. 40, no.3, pp. 613-627, 1995.